SCHOOL MATH PROBLEMS WITH REAL LIFE APPLICATIONS

Victor Rabinovich
Website: matholympus.com

Please contact the author, Victor Rabinovich, at vicrabinovich@gmail.com to suggest corrections and additions

Copyright © 2015 Victor Rabinovich

All rights reserved.

ISBN: 978-0-9940481-0-3

Printed in Canada
To my wife Irina Rabinovich, my children

Maria, Elena, Ekaterina, Dmitri,

And to my grandchildren
Table of Contents

Contents

Introduction ................................................................. vii

Part 1                  Problems

1: Geometry................................................................. 1

2: Percentage and Finance........................................... 23

3: Bar and Pie Graphs.................................................... 41

4: Logic................................................................. 67

5: Linear and Circular Motion................................. 83

6: Math for Physics..................................................... 107

7: Probability and Statistics....................................... 127

8: Work Word Problems............................................. 141

Part II                  Solutions and Answers

9: Geometry................................................................. 143

10: Percentage and Finance................................. 167

11: Bar and Pie Graphs.................................................... 179

12: Logic................................................................. 189

13: Linear and Circular Motion................................. 203

14: Math for Physics..................................................... 223

15: Probability and Statistics................................... 245
Table of Contents

16: Work Word Problems................................. 255

About the Author and Acknowledgments..........259
Introduction

We are faced with math all around us! Math is needed in everyday life, such as at the store and at the factory, at the bank and in a research laboratory, etc. A strong understanding of mathematical concepts can help students with punctuality, to deposit money into a bank account, with purchasing a home, shopping for groceries, flying on a plane, putting fuel in the car, paying the bills etc. Therefore, it is crucial for young students to develop mathematical skills that they can apply to solve problems of everyday life.

Our book presents a collection of math problems for students of 12 years and older, who seek to improve and to deepen their level of understanding in mathematics. This workbook is geared toward learning and developing mathematical concepts through practice. It consists of approximately 400 fundamental and advanced mathematical problems grouped into the following 8 categories:

1. Geometry;
2. Percentage and Finance;
3. Bar and Pie Graphs;
4. Logic;
5. Linear and Circular Motion;
6. Math in Physics;
7. Probability and Statistics;

The various math examples, problems, practice questions, and tests are taken from various disciplines, including physics, electronics, and economics. We would like to show through the exercises that math is not an abstract discipline; for most people math is an instrument that is applicable to everyday living.

This workbook includes both fundamental and advanced practice, with real life applications. The questions range in degree of difficulty, and include either detailed solutions or simple answers at the end of the book. The solutions in the back allow students to work alone if a tutor is not available. This workbook easily supplements the math
Introduction

curriculum of middle and high school courses, which enables students to use it year round outside of class instruction.

This workbook was created with tutors and teachers in mind, as we strive to provide effective and stimulating practice opportunities for students.

This workbook is also very useful for those students preparing to write the PISA test, which evaluates education systems worldwide by testing 15-year olds in key subject areas, including math. According to recent test results, published at the end of 2013, Canadian and American students did well, but not as strong as students from Asian countries, Australia, or Finland.

The problems in this workbook are based on different resources; including North American ideas about teaching mathematics, Russian traditional approaches to learning math, and the experience of the authors (who designed many original questions specifically suited for this age group).

There are numerous approaches taken to teaching and understanding math concepts in different places around the world. Below is the break-down of the combinations of approaches taken by the authors of this workbook:

The North-American approach:
- significant emphasis on applying the study of mathematics to real-life situations
- important role of visual material in the learning/teaching process
- strong focus on the study of fundamental math skills using mainstream questions/problems
- learning the same mathematical concepts with an increasing complexity from elementary to high school

Russian traditional teaching system includes:
- strong emphasis on daily practice/homework
- teacher directed learning and instruction
- high level of difficulty beginning in elementary school
Introduction

The successful combination of different methodological techniques, like those mentioned above, undoubtedly help to improve the level of understanding of mathematics taught at schools.

Our book collects math problems that covers a wide range of different topics devoted to solving math problems with real life applications for students aged 12 years and older.
Introduction
Chapter 1

Geometry

Problems

1.1. A television screen is 100 cm diagonally, and the ratio of the television screen height to the width is 3:4. Determine the height and width of the TV screen.

1.2. The figure shows a podium to honor athletes. The widths of all three steps have the same length. What is the area of the front side surface? Dimensions are in millimeters.
1.3. A building lot has a quadrilateral shape, and its lengths are in the ratio 3:4:5:6. Find all lengths if the perimeter is equal to 540 m.

1.4. A building lot has a quadrilateral shape, and its angles are in the ratio 3:4:5:6. Find all of the angles of the quadrilateral lot.

1.5. A right triangle–shaped land ABC is divided by an altitude AD to the hypotenuse BC into two pieces, as shown in the attached figure. Find the perimeters of the triangular lots ABD and ADC if AB=120 m, AC=160 m.

1.6. A right triangle land ABC is divided by altitude AD into two triangles, as shown in the attached figure. Find the ratio of the area
S₁ of triangle ABD to the area S₂ of the triangle ADC if AB=120 m, AC=160 m.

1.7. The side lengths of a triangle property lot are equal to 20, 21, and 29 m. Find the lot area.

1.8. A building lot has the shape shown in the attached figure. Find the area of the lot if the area of the colored unit is 1 m².

1.9. What is the perimeter of a rectangular building lot with an area of 2,000 m² if one side is five times longer than the other one? Give the answer in meters.
1.10. What are the side lengths of a rectangular lot, if they are in proportion 3:5, and the area of the rectangular is equal to 135 m²?

![Rectangle](image)

135 m²

1.11. What are the sides of a rectangular building lot if one side is 6 m greater than the other, and the area is 135 m²?

1.12. Find the side lengths of a rectangular piece of land if its perimeter is 360 meters, and its area is equal to 8,000 m².

1.13. The attached figure shows two triangular property lots ABC and BDC. Lot ABC has the form of an equilateral triangle, while lot BDC is an isosceles triangle with equal sides BD=BC. Determine the ratio of these triangles’ areas.

1.14. A lot in the shape of a right triangle is divided into two parts by an altitude drawn from the vertex with the right angle to the hypotenuse. Find the area of each part if the altitude value is 12 cm, and the perimeter of the original right triangle is equal to 60 cm.
1.15. A triangular property lot is bounded by the following lines: $y = \frac{x}{2}$, $y = x - 2$, and the axis $Y$. Define the area of the resulting triangular lot.

1.16. A property lot is the sum of five identical squares, as shown in the attached figure. How can this be divided into two parts of equal area using one straight line?

1.17. It is necessary to make 6 identical timbers with equal cross-section squares from one original round log. The length of each timber has to be the same as the length of the original round log. Suggest a solution and calculate the side length of one cross section square if the radius of the round log is $R$. 
1.18. It is necessary to make 6 identical timbers with equal cross-section squares from one original round log (problem 1.17). The length of each timber has to be the same as the length of the original round log. Suggest a solution and calculate the area of one cross-section area if the radius of the round log is equal to 15 cm.

1.19. On a blackboard, a teacher drew a land lot in a shape of an isosceles trapezoid with three equal sides: AB=BC=CD=1. Prove that the area of the trapezoid is maximal if AD=2 (AE=FD=1/2).

(Hint: Evaluation of the square root: $\sqrt{1+x} \approx 1 + \frac{x}{2}$ if $x << 1$).

1.20. How many boxes with a rectangular parallelepiped shape with dimensions $20 \times 40 \times 80$ (cm$^3$) will fit on a truck with dimensions $2.4 \times 8 \times 3$ (m$^3$)?
1.21. How many rectangular tiles will be required to cover the rectangular area of 2 m², if a tile’s diagonal length is 50 cm² and the tiles’ rectangular lengths are integer numbers?

1.22. A 1.7 m tall man stands at some distance from the street light DC. How high (in meters) is the street light if the ratio of the distance (AC) between the street light and the man to the shadow (SA) of the man is equal to 2?

1.23. A ladder with a length of 5 m is placed against a wall. At what height from the ground is the upper end of the ladder if the height value is an integer number?

1.24. The width of a soccer gate net is 2BC=7 m and it has a height of CD=5.5 m. A player standing at a distance of AB=11 m from the gate as shown in the attached figure, wants to get the ball into the top corner. What is the value of the distance AD shown in the figure by the thick black line?
1.25. The width of a soccer gate is $2BC=7$ m and the height is $CD=5.5$ meters. The player standing at a distance of $AB=4.2$ m from the gate, as shown in the attached figure, wants to get the ball into the top corner. At what angle to the horizon should the ball be kicked to get into the top corner?

1.26. The ski track is a closed smooth loop on a flat surface. The distance between an athlete’s right and left skis is equal to 0.15 m. What is the difference $\Delta$ between the distances traveled by the right and the left feet of the skier?

1.27. A truck with a width of 2 m has to pass under the bridge in the shape of a semicircle with a radius of 4 m. What should the maximum height of the truck be if it does not touch the bridge? In the response, estimate the approximate value to the nearest tenth.

1.28. The area of Belarus is 208,000 km$^2$. Forty-seven percent of the territory is agricultural land, while water surface area is six times
smaller than forested area and uncultivated land is twice the size of the water surface area. How big is the forested area?

1.29. The building has a gable roof with an inclination from one side of $30^\circ$ and $60^\circ$ from the other, as shown in the attached figure. Find the sum of the lengths $AB+BC$ if $BD=10$ m.

1.30. The roof of a hangar is a part of a cylinder, as shown in the attached figure (left). The cross-sectional area of the hangar is half of the total circular area (right). Find the area of the hangar roof if the diameter of the circle in the cross-section is equal to $D=12$ m, and the length of the hangar is $L=25$ m ($\pi=3.14$).
1.31. The roof of a hangar is part of a cylinder, as shown in the attached figure (left). Find the cross-sectional area ABC of the hangar, shown as a shaded in the attached figure (right), with an angle \( \angle AOC=90^0 \) and radius \( OC=6 \) m.

1.32. The cross-section of the garage hangar is part of a parabola. The hangar height \( h=20 \) m, while the width of the hangar \( L=10 \) m. Define constants \( A \) and \( B \) for the parabolic (quadratic) function \( Y = A \times X^2 + B \).

1.33. The cross-section of the garage hangar is part of a parabola \( Y = -\frac{4}{5} \times X^2 + 20 \). Define the width \( L \) and the height \( h \) of the hangar.
1.34. A steel beam with a length of 20 m is fixed at two ends A and C, as shown in the attached figure. The thermal heating of the beam length (dash dotted line) is increased by an amount equal to 0.02 m. Make an approximate estimation of the curvature radius OC=OA of the beam after heating. (Hint: Present arc ABC in the form of two lines AB and BC).

1.35. From point A, mountain peak C is visible at an angle of 30°. From point B, the mountain peak becomes visible at an angle of 45°. The distance between points A and B is equal to 1,000 m. Find the approximate height of the mountain in meters.

1.36. From point A, mountain peak C is visible at an angle of 30°. From point B, the mountain peak becomes visible at an angle of 60°. The distance between points A and B is equal to 1,000 m. Find the approximate height of the mountain in meters.
1.37. From the top of the first mountain AB, the peak of the second mountain DC is seen at an angle of $30^0$; meanwhile, from the foot of the first mountain, the top of the second mountain is visible at an angle of $60^0$. Determine the height DC of the second mountain and the distance AD between the first and second mountains if the height AB is equal to 100 m.

1.38. The level of the liquid in the cylindrical container is equal to $h = R$, where R is the radius of a cylinder’s base circle. The level of the same amount of liquid in the cube-shaped container with a side $2R$ is equal to X. What is the ratio $R/X$?

1.39. The attached figure (left) shows a conical container with water in it. A base radius of the container is R and a height is equal to L. At what height H relative to L will the water level be if all of the water from the conical container is poured into a cylindrical tank (right
1.40. It is necessary to build a cube-shaped tank with a capacity for 64 liters of water. How many square meters of the material would be required to make such a cube?

1.41. What is the volume of the cornice with a length of 1 m? (A cornice, or “ledge”, is generally any horizontal decorative molding that crowns a building or furniture element.) The cross-section shape of the cornice is shown in the figure below. ED=3 cm, radius FE=4 cm, radius OS=5 cm, side BC=16 cm, AB=3 cm.

1.42. A tank for liquid consists of a hemisphere with radius of 0.25 m and a cylinder with the same base radius. How high should the cylindrical part be if the whole volume of the tank is equal to 0.25 m$^3$?
1.43. A container for liquid consists of a hemisphere of radius $R$ and a cylinder with the same base radius. At what ratio between the radius $R$ and the height of the cylinder $h$ is the volume of liquid in the cylindrical portion equal to the volume of the liquid in the hemisphere?

1.44. A crystal has the shape of two regular quadrangular pyramids connected by their base planes. A side of the common base is 3.5 cm long, and the distance between the tops of the pyramids are equal to 5 cm. Find the volume of the crystal.
1.45. Find the volume of the shape shown in attached figure if the diameters of the horizontal and vertical cylinder parts are equal to 5 cm (π≈3.14).

1.46. A cylindrical water tank has an inner diameter $D_1$ equal to 1 m, an outer diameter $D_2 = 1.1$ m, and a height $H = 3$ m. How much more paint is needed to paint the outer surface of the tank than the inner surface if 1 m$^2$ requires $Q=100$ g of paint?

1.47. One of the most spectacular ancient structures, the pyramid of Cheops has the shape of a square pyramid with a height $OS$ of 146.7 m and a lateral edge $AS = 219.2$ m. Find the total lateral surface area and volume of the pyramid.

1.48. If a square pyramid is sliced by a plane EFGM parallel to the base ABCD of the pyramid and a plane bisects the altitude OK of the
pyramid, what percentage of the volume of the pyramid (the shaded part in the attached figure) lies below the slicing plane? Keep in mind that ON=NK.

1.49. A first cylindrical cup with a diameter of $D_1=8$ cm and a height of $h_1=10$ cm has a capacity of $V_1=0.5$ l of water. Find the dimensions ($D_2$ and $h_2$) of a second cup, which has a similar geometry to that of the first cup, if the second cup has a capacity of $V_2=4$ liters of water?

1.50. A vehicle mass is equal to $M_1=1,050$ kg. A similar model car is made with a linear scale of 1 to 60. Determine the mass of the vehicle model $M_2$ if it is made from the same material as the car itself.
1.51. Paper tape is wound tightly around the outside of a uniform solid cylinder with a diameter to 20 cm. The paper’s thickness is 0.5 mm and the thickness of the roll is 10 cm. Find the length of the paper tape. Express your answer in meters, assuming that $\pi=3$.

![Cylinder Diagram]

1.52. The diameter of the Moon is (approximately) a quarter of the Earth’s diameter. Determine the ratio $q$ of the volume of the Earth to that of the Moon, considering both of them as spherical shapes.

![Earth and Moon Diagram]

1.53. If the surface area of the planet Mercury increases 2 times with respect to a nominal value, its volume will be 6.4 times less than the volume of the Earth. Determine the ratio of the radius of the planet Mercury to the radius of the planet Earth. Assume in your calculations that both Earth and Mercury are spherical.

![Planets Diagram]
1.54. How far can the point T located on the earth surface be seen from a hot air balloon if it is located 4 km above the earth? (The radius of the earth is around 6,370 km.)

1.55. How many degrees does the earth rotate around its own axis in 4 hours?

1.56. How many degrees does the earth orbit around the sun in 4 hours?
1.57. Determine the Earth’s radius if 1 m is a 40-millionth of the equator length.

1.58. Assume that the Earth along the equator is a circle with a length of \( \approx 40,000 \) km. The length of the arc between two points A and B on the Earth along the equator \( \cap \angle ACB = 6,667 \) km. What is the shortest distance between points A and B, defined by a chord, as shown in the attached figure?

1.59. The Moon diameter is seen from the Earth at an angle of 30 '. Find the approximate distance to the Moon, knowing that its diameter is approximately equal to 3,400 km (\( \pi \approx 3 \)).
1.60. The length of the Earth’s equator on the globe is equal to 1 m. What is the area of Russia on the globe if the actual length of the Earth’s equator is 40,000 km and the area of Russia 17,000,000 km$^2$?

1.61. The length of the Earth’s equator on the globe is equal to 1 m. What is the area of Canada on the globe if the actual length of the Earth’s equator is 40,000 km and the area of Canada is 10,000,000 km$^2$?

1.62. The Earth’s surface has a spherical shape; the length of a great circle $L$ is approximately equal 40,000 km. Part of the boundary between Canada and the United States is located at a latitude of about 45$^0$. Determine the circumference of the $R$ around the Earth, passing through this latitude.
1.63. Suppose you tie a rope tightly around the Earth’s equator. You add an extra 1 m to the length. If the extra rope is distributed evenly around the globe, will there be enough space between the rope and the surface of the earth for a mouse to crawl under?

1.64. Determine how many Earth days it takes for the Moon to orbit the Earth. All parameters required to make the calculation are shown in the attached figure.
Chapter 2

Percentage and Finance

Problems

2.1. A family purchased a home two years ago for $100,000. The price of a house rises by 2% per year. How much is this house worth today?

2.2. The price of sugar increases by 10% per kilogram. Now you can buy less sugar for the same amount of money. Find the ratio $A/B$, where $A$ is the initial weight of the sugar and $B$ is the weight of the sugar after the price increase that can be bought for the same amount of money.
2.3. Daniel sells a toy for $100 after a 20% discount. What was the original selling price?

2.4. On January 1st, the power meter reading was 74,958 kilowatt-hours; on February 1st, it was 800 kilowatt-hours more than on January 1st; and on March 1st, it was 400 kilowatt-hours less than on February first. How much should you pay for the electricity for January plus February, if 1 kilowatt-hour costs 10 cents?

2.5. Find the perimeter of a rectangular lot if its length is equal to 520 m and its width is 70% shorter than its length.

2.6. Find the area of a rectangular lot if its length is equal to 520 m and its width is 70% shorter than its length. Answer in hectares.

2.7. An employee’s salary is $2,000 per month. Because of a problem in the company, his salary was temporarily reduced by 20%, and
2.8. Increasing $100 by a certain percent produces the same result as decreasing $300 by the same percent. What is this percent?

2.9. Last year, the company had 2,000 units of sales at $160 per unit. In the current year, the marketing manager projects a 25% increase in unit volume with a 10% price increase. Returned merchandise should represent 5% of total sales. What is the net dollar sales projection for this year?

2.10. Student group chooses a volleyball team captain. Ten children voted for Peter, 14 for Victor, 4 for Alexander, and 12 for Shawn. Assume that the same student cannot vote twice. What percent of children have voted for Victor and Shawn together?

2.11. During the year, an employee’s salary is increased by 32%, and in the first half of the year, it is increased by 10%. By what percent has the salary increased for the second half of the year?
2.12. What is the initial amount of money in an investment account if the annual rate is equal to 4% and the sum has increased to $10,816 over two years?

2.13. A farmer sells two types of watermelons: The first one costs $10 each, and has a very thin rind, while the second has the same volume $V$ and costs $7 each, but 20% of its volume is rind. Which type of watermelon is preferable to buy?

2.14. The speed of one printer is 30% greater than that of a second printer. If the first printer prints 150 text pages how many pages will the second one print for the same period of time?

2.15. The price $A$ of one kilogram of apples is 60% more than the price $P$ of 1 kg of pears, while the price $P$ is 20% less than the price $G$ of 1 kg of grapes. What is the ratio of $A/P$? Answer as the ratio of integer digits.

2.16. A scanner that cost $400 is now on the sale with an 8.5% discount. How much does the scanner cost with the discount?
2.17. Victor has a net income of $1,240 per month. If he spends $150 on food, $244 on his car payment, $300 on rent, and $50 on savings, what percent of his net income can he spend on other things?

2.18. Twelve parents did not attend a school parents’ conference, representing 7.5% of the total number that were invited. How many parents were invited to the conference?

2.19. February store sales increased by 20% in comparison with January, and in March, the store sold 20% more goods than in February. What is the percent of increase from the original January price to the final March price?

2.20. Over 10 months, from the beginning of January to the end of October, store sales rose by 5% each month in comparison with the previous one. What was the percent increase of the total price from January to October?

2.21. In a Canadian city, residents speak English and French. In fact, 80% of the residents speak English and 80% of them speak French. What percentage of the residents are bilingual and can speak both English and French?
2.22. There are two kinds of milk: 1,000 g with 2% of fat and 1,000 g with 18% of fat. How many grams of milk first and second types must be mixed to obtain 1,000 g of milk with a fat content of 5%?

2.23. A first alloy contains 10% copper, while a second alloy contains 40% copper. The weight of the second alloy is 3 kg more than that of the first. A third composition built from these two alloys contains 30% copper. Find the mass of the third alloy in kilograms.

2.24. Ten kilograms of an alloy contain 40% copper and 60% steel. How many kg of copper should be added to the alloy to get equal percentages of steel and copper?

2.25. A tank with 4 l of a mixture contains 95% water and 5% alcohol. What is the percent of the alcohol in the new solution if the same tank contains one additional liter of water?

2.26. A chemist has 30% and 50% acid solutions. What amount of each solution should be used in order to make 300 ml of a solution with 40% acid content?

2.27. The company bought a car on sale 35% below the regular price, and sold it at a price that was 50% higher than the purchase price. Did the company make a profit in comparison with the original price of the car?

2.28. Victor’s contribution to the bank account is equal to $1,000. If the bank pays 3% interest annually, how much money will be in his account after 10 years?
2.29. Victor invested money at a 5% interest rate annually for 10 years. Shawn invested money at an interest rate of 7% for 8 years. What is the ratio of Victor’s initial contribution to the initial Shawn contribution if they receive the same final amount of interest?

2.30. Peter borrowed $800 at 10% per year, for 2 years. How much does he have to repay (principal + interest) at the end of the 2-year period?

2.31. John borrows $1,000 for 2 years from Bank A, whose annual interest rate is 9%. Furthermore, John will repay the loan in two equal payments at the end of each year. Find the annual payment.

2.32. Mary purchases furniture for $4,200. She decides to finance the furniture for 2 years with a loan. The loan requires a 12% down payment and 24 equal monthly payments of $195. What will Mary’s finance charge for her loan be?

2.33. A 10 kg watermelon is made up of 99% water. We let the watermelon sit out in the sun, and some water evaporates. Now the watermelon is only 98% water by weight. What does the watermelon now weigh?

2.34. Fresh mushrooms are 90% water. Dried mushrooms are only 12% water. How many kilograms of dried mushrooms do we get from 11 kg of fresh ones?
2.35. Five baskets and two suitcases weigh half of the weight of three baskets and ten suitcases. What percent of the basket weight in the weight of the one suitcase?

2.36. A home buyer took a bank loan of $400,000 at a year interest rate of 3% for 25 years. Estimate how much money the buyer will return to the bank in 25 years if the monthly payment $M$ for a loan of P dollars for n months at monthly rate $I$ is

$$M = \frac{P i}{1 - (1 + i)^{-n}}.$$  

When calculating, monthly payment round to the nearest to the nearest one.

2.37. The boys weighed a bag of fruit, and found that it was 12.3 kg. Pears weigh 1.5 times more than apples, while oranges weigh 60% more than apples. What do apples, pears, and oranges weigh?

2.38. Estimate the area of a leather piece that will be used to build a soccer ball with a radius of 10 cm, given that the seams will leave an extra 8%.

2.39. The dollar exchange rate is up by 10% in comparison with the Russian ruble. How much has the ruble fallen in percentage?
2.40. The number of girls in the classroom is 60% of the number of boys. What percent of all students are girls?

2.41. A family consists of two people—a husband and his wife. If the wife’s salary is doubled, the total family income will increase by 45%. Let’s assume that the husband’s salary is doubled. Find the percent of the total family income.

2.42. Peter’s bottle has a water volume which is 10% greater than the water volume of Greg’s bottle. Peter takes a sip from his bottle of water representing 11% of the original volume, and Greg takes a sip representing 2% from his own bottle. Who has more water now?

2.43. A store sells different sizes of pizza: a pizza with a radius of 30 cm for $18, one with a radius of 25 cm for $10, one with a radius of 20 cm for $7, and one with a radius of 15 cm for $5. Which pizza size would you prefer to buy?
2.44. A person slices a pie into $k$ equal pieces and eats one piece. In terms of $k$, what percent of the pie is left?

2.45. A home owner pays $120 per month (30 days) to heat his house when the monthly average outside temperature is $-10^\circ$C. With an average outside temperature equal to $-20^\circ$, the owner has to pay $200 per month. Estimate (up to one decimal place) the cost of heating the house over 8 hours at an average outside temperature of $-15^\circ$, if the charge is a linear function of the temperature and heating time.

2.46. A car with a gas engine costs $30,000. A car of the same class with an electric engine costs $40,000. The cost of gas is increasing each year by 5%, starting at $1,500 in the first year. The cost to recharge the battery for the electric car increases every year by 10%, starting from $500. Which car is more economical if both run for 10 years with an annual mileage of 15,000 km? Ignore repair costs, but take into account the car prices.

2.47. A family is going to buy a car to drive in the city and out of town. They don’t know which car is preferable—a $20,000 car with a gas engine or a $25,000 car with a diesel engine. They know that the monthly mileage estimation is as follows: 1,800 km in the town and 600 km in the country. A liter of gasoline costs $1.20, while a liter of diesel costs $1.40. Gas consumption per 100 km in the city is 10 l, while out of town it is 8 l. For the diesel engine, consumption is 8 l in the city and 5 l out of the town. Which car will be more cost efficient after 5 years usage. Take into account the cost of purchasing the car and fuel expenses.
2.48. A family is going to buy a car to drive in the city and out of town. They don’t know which car is preferable—a car with a gas engine or a car with a diesel engine. They know that monthly mileage estimation is 1,800 km in the town and 600 km in the country. A liter of gasoline costs $1.20, while a liter of diesel costs $1.40. Gas consumption per 100 km in the city is 10 l, while out of town it is 8 l. For the diesel engine, the consumption is 8 l in the city and 6 l out of the town. Do you think it is preferable to buy the diesel car if it is known that a car with gas engine is cheaper than one with a diesel engine? Take into account that family will use the car over 5 years.

2.49. A family is going to buy a car to drive in the city and out of town. They don’t know which car is preferable—a gas car or a diesel car. They assume that the monthly mileage is 2,000 km in town. A liter of gasoline costs $1.20 and a liter of diesel costs $1.40. Gas consumption per 100 km in the city is 10 l, while out of town it is 8 l. For the diesel engine, the consumption is 8 l in the city and 5 l out of town per 100 km. For what mileage of out of town will the expenses for a gas car be less than those for a diesel car? Take into account that the family will use the car over 5 years, and that the gas car costs $20,000 while the diesel car costs $25,000.

2.50. In a car dealership, there are two cars displayed: The first runs on gasoline and costs $20,000, while the second runs on diesel and costs $25,000. Gasoline costs $1.20/l and diesel costs $1.40/l. The first car will utilize 10 l of gasoline per 100 km driven, while the second will use 7 l of diesel. Which car is more cost efficient if it will be driven for 300,000 km? The cost of vehicle maintenance does not need to be considered in your calculations.
2.51. The first seller has goods worth $100,000, and the second has goods worth $55,000. Every day, the first seller sells goods worth half the amount remaining from the previous day, while the second seller has goods worth $15,000 less than the previous day. Determine how many days must pass before both sellers’ goods are worth the same amount, if it is known that the remaining dollar value is a positive integer.

2.52. Daniel decides to buy a car for $20,000. He and his sister (together) have $28,000 in their bank accounts. The sister has $4,000 less than her brother. Is there enough money to buy a car if the sister is willing to lend Daniel $3,000?

2.53. It is necessary to carry 40 tons of cargo to a distance of 1,000 km. Four companies offer transportation service. The cost of transportation to a distance of 100 km and the capacity of the vehicle is shown in the table below. Find the cheapest transportation company.

<table>
<thead>
<tr>
<th>Company</th>
<th>Cost of 100 km transportation</th>
<th>Truck capacity (tons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>$65</td>
<td>4</td>
</tr>
<tr>
<td>#2</td>
<td>$40</td>
<td>2.5</td>
</tr>
<tr>
<td>#3</td>
<td>$50</td>
<td>2</td>
</tr>
<tr>
<td>#4</td>
<td>$25</td>
<td>1.5</td>
</tr>
</tbody>
</table>

2.54. A store sells 15 different kinds of sausages, and each type is worth an integer number of dollars. Can the average price for all varieties of sausage be equal to $10.15?

2.55. You visit Staples and decide to buy 4 identical rolls of tape, 8 identical notebooks, one book for $7, and 2 pens. The seller
estimates the full purchase price and asked you to pay $12.27. Do you think he made a mistake or not?

2.56. The store has bags of flour weighing 16, 17, and 40 kg. Without opening any bag, can I combine these bags to obtain a weight equal to 140 kg?

2.57. Victor wants to buy 2.5 kg of sweets in red and blue packs. Each red pack weighs 450 g and each blue one weighs 100 g. How many red and blue packs does he need to makeup 2.5 kg?

2.58. After the farmer plows 84% of the field, he has to plow another 8 ha. What is the area of the whole field?

2.59. A construction company plans to buy 25 tons of laminate from one of three suppliers. The weight of one pack of laminate is 5 kg. The prices and terms of delivery are given in the table. What is the cheapest option?

<table>
<thead>
<tr>
<th>Supplier</th>
<th>Price for 1 pack</th>
<th>Delivery price</th>
<th>Delivery conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$5.1</td>
<td>$1000</td>
<td>---</td>
</tr>
<tr>
<td>B</td>
<td>$5.2</td>
<td>$400</td>
<td>---</td>
</tr>
<tr>
<td>C</td>
<td>$5.4</td>
<td>$200</td>
<td>50% discount if the order is more than $26,000</td>
</tr>
</tbody>
</table>
2.60. A telephone company offers customers three different monthly plans, as shown in the following table.

<table>
<thead>
<tr>
<th>Plan</th>
<th>Payment</th>
<th>Payment for 1 minute of conversation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time-based</td>
<td>---</td>
<td>15 cents</td>
</tr>
<tr>
<td>Combo</td>
<td>$20 for the first 600 minutes</td>
<td>25 cents after 600 minutes</td>
</tr>
<tr>
<td>Unlimited</td>
<td>$250</td>
<td>---</td>
</tr>
</tbody>
</table>

A customer uses 1,000 minutes per month. Which plan is the most cost-effective?

2.61. A telephone company offers customers three different monthly plans, as shown in the following table.

<table>
<thead>
<tr>
<th>Plan</th>
<th>Payment</th>
<th>Payment for 1 minute of conversation</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1 Time-based</td>
<td>---</td>
<td>15 cents</td>
</tr>
<tr>
<td>#2 Combo</td>
<td>$20 for the first 600 minutes</td>
<td>25 cents after 600 minutes</td>
</tr>
<tr>
<td>#3 Unlimited</td>
<td>$250</td>
<td>---</td>
</tr>
</tbody>
</table>

At what number of call minutes does plan #1, or #2, or #3 become the most cost effective?

2.62. An internet service provider offers the monthly customer plans for cell phones in the following table.

<table>
<thead>
<tr>
<th>Usage (Mb)</th>
<th>Price per Month</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–100</td>
<td>$10</td>
</tr>
<tr>
<td>0–500</td>
<td>$30</td>
</tr>
<tr>
<td>0–2,000</td>
<td>$80</td>
</tr>
<tr>
<td>0–6,000</td>
<td>$90</td>
</tr>
<tr>
<td>0–10,000</td>
<td>$100</td>
</tr>
</tbody>
</table>
You can order any of these plans. However, you have to take into account that if the megabyte limit is exceeded, it will be necessary to pay the following fees:

a. If the excess is in the range of 0–100 Megabytes (Mb) per month, you pay an additional $10;
b. If the excess is in the range of 100–200 Mb per month, you pay an additional $20;
c. If the excess is in the range of 200–300 Mb month, you pay $30, and so on.

Which plan is most cost effective for the year, if you use 600 Mb per month during the first 10 months, and –1,300 Mb in November and December?

2.63. An internet service provider offers the following monthly customer plans for cell phones:

<table>
<thead>
<tr>
<th>Usage (Mb)</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–100</td>
<td>$10</td>
</tr>
<tr>
<td>0–500</td>
<td>$30</td>
</tr>
<tr>
<td>0–2,000</td>
<td>$70</td>
</tr>
<tr>
<td>0–6,000</td>
<td>$90</td>
</tr>
<tr>
<td>0–10,000</td>
<td>$100</td>
</tr>
</tbody>
</table>

You can order any of these plans. However, you have to take into account that if the megabyte limit is exceeded, it will be necessary to pay the following fees:

a. If the excess is in the range of 0–100 Megabytes (Mb) per month, you pay an additional $10;
b. If the excess is in the range of 100–200 Mb per month, you pay an additional $20;
c. If the excess is in the range of 200–300 Mb month, you pay $30, and so on.

Which plan should be chosen for the year if you use 600 Mb per during the first nine months, and –1300 Mb in October, November and December?

2.64. A store sells two jars of strawberry jam. One jar is twice as tall as the other, but twice as small in diameter. The taller jar costs $1,
and the other one costs $1.5. Which jar is a more cost-effective purchase? Assume that jar has cylindrical shape.

2.65. A farmer decides to use a land area of 100 ha to plant corn and sugar beets. It is known that the corn crop from 1 ha is equal to 50 tons, while the sugar beet crop yields 20 tons. The farmer needs to obtain 3,200 tons of crop from the total area. What is the smallest area that can be planted with corn?

2.66. There are two identical bottles: The first contains a mixture of 9 l of water and 1 l of paint, while the second contains 5 l of water. Five liters of the mixture from the first container is transfused into the bottle with 5 l of water. Then, 5 l of the stirred mixture from the second bottle is transfused into the first bottle. Finally, 5 l of the stirred liquid from the first container is transfused into the second container. Find the final percentage of paint in the first and second bottles after these transfusions.

2.67. There are four identical bottles. The first one contains a mixture of 9 l of water and 1 l of paint that is fully mixed with the water. The second bottle contains 5 l of water, the third contains 2.5 l of water, and the fourth contains 1.25 l of water. Half of the mixture from the first bottle is transfused into the second bottle. Then, half of the mixture from the second bottle is transfused into the third bottle. Finally, half of the mixture from the third bottle is transfused into the fourth bottle. What is the percentage of paint in the fourth bottle?

2.68. During first and second half of the year, a company increased production of electronic components by the same percentage. Find this number if it is known that at the end of the year, 1.21 times more
electronic components were produced than at the beginning of the year.

2.69. The initial authorized capital of the company increases by 25% (due to profit) each year. Estimate the minimum number of years it would take to increase the initial capital of the company by more than three times.

2.70. Four identical pairs of shoes in total cost less than one pair of boots by 8%. In terms of percentage, how much more expensive are five pairs of shoes in comparison with one pair of boots?

2.71. The distance between plants A and B is 40 km. The demand for oil at plant A is 80 tons per day, while that at plant B is 70 tons. Transporting 1 ton of oil over 1 km costs $80 for plant A, while it costs $100 for plant B. Where a tank farm should be built (between plant A and B) to provide fuel to plants A and B with minimal transportation fees?
Chapter 3

Bar and Pie Graphs

Problems

3.1. Nutritionists recommend the following proportion for the total daily food intake: 25% morning breakfast, 15% second breakfast, 45% lunch, and 15% dinner. Build a bar chart showing this proportion. (You can use Microsoft Word or Excel.)

3.2. Nutritionists recommend the following proportion for the total daily food intake: 25% morning breakfast, 15% second breakfast, 45% lunch, and 15% dinner. Build a pie chart showing this proportion. (You can use Microsoft Word or Excel.)

3.3. Create a pie chart that defines the distribution (percentage) of various areas of the oceans around the globe.

The world consists of four oceans:

- Pacific: 179.7 million km²
- Atlantic: 93.4 million km²
- Indian: 74.9 million km²
- Arctic: 13.1 million km²

3.4. Create a pie chart for the distribution of Earth’s landmass in percentage if:

- Forest: 57 million km²
- Steppe: 24 million km²
- Tundra, deserts, marsh: 54 million km²
• Arable land: 15 million km²

3.5. The table shows the number of students in the classroom who came from different countries of the world.

<table>
<thead>
<tr>
<th></th>
<th>Russia</th>
<th>USA</th>
<th>Great Brittan</th>
<th>Canada</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
<td>50</td>
<td>10</td>
<td>25</td>
</tr>
</tbody>
</table>

 Which of the diagrams accurately reflects the data in the table?

3.6. 90 students have taken a math test. 30 people’s scores were rated good, and 20 students’ scores were rated excellent. The number of students who did not pass an exam is three times less than those who did. Which of the following diagrams correctly reflects the results of the exam?

I  II  III  IV
Bar and Pie Graphs (Problems)

3.7. The pie chart shows the results of a math test, which was taken by 400 students. Determine the number of students who received scores of excellent, good, pass, and fail.

3.8. The pie chart shows the results of a math test. Determine the number of students who received scores of excellent, good, pass, and fail.

3.9. The pie chart shows the results of a math test. Determine the number of students who received scores of excellent, good, pass, and fail if the number of the students with excellent and fail scores is equal to 325 and the number of students who received good scores is 45% greater than the pupils with fail scores.
3.10. A diagram in the figure below shows the number of sales in Store #1 and #2 during four months. Which store had more sales from January to April?

3.11. A bar chart in the figure below shows the number of sales of identical toys in two different stores (#1 and #2) during four months. One toy price is equal to seven dollars in Store #1 and five dollars in Store #2. Which store has more income?
3.12. What should be the number of sales in Store #1 if the ratio of total sales during four months in Store #2 to #1 is equal to 1.5?

![Bar graph](image)

3.13. Two brothers opened stores that sell electronics. The bar graph shows sales (in percentage) for both stores during four months (January sales are taken as 100% in both stores). However, the first store graph shows sales change in comparison with January, and the second store graph shows sales change in comparison with the previous month. Which of the stores has more profit after four months of work?

![Bar graph](image)

3.14. The profit of the store during fifteen years is expressed with the help of the bar graph below. Find the profit of the store during 12-th year and the total profit after 12 years.
3.15. Four candidates participate in the presidential elections. The preliminary survey indicates the following distribution: the first candidate received 28% of the vote, the second candidate received 32% of the vote, the third candidate received 80% of the votes that received candidate #2, and the fourth candidate received the rest of the votes. Build a pie chart that demonstrates the relationship between the percentages of votes for all candidates.

3.16. Four candidates participate in an election. The preliminary survey indicates the following distribution: the first candidate received 28% of the vote, the second candidate received 25% of the vote, the third candidate received 20% more votes than candidate #2, and the fourth candidate received the rest of the votes. Build a pie chart that demonstrates the relationship between the percentages of votes of all candidates.

3.17. The table and bar chart below show the results of the math test in class A and class B. The horizontal axis on the bar chart represents the number of students that received a particular score, on the vertical axis—the score value. Find the average score for class A and class B. Which score is higher?
The graph below shows test results of several student groups. Group A includes 35 students, group B has 30 students, group C comprises 25 people, and there are 40 students in group D. Using the data and bar graph, determine the average percentage of students with excellent, good, and satisfactory grades.
3.19. The bar graph below shows the number $N$ of tickets that were sold in the theatre during one week. Find all combinations of possible days that correspond with ticket sales equal to 1,800. For example, 600 tickets were sold for days #1 and #4.

![Bar Graph Example](image)

3.20. Gas consumption (town and country) per 100 km for four cars of different companies is shown in the bar graph. Assuming that the annual average of kilometers driven per year in the city and in the country is the same, determine which car is more economical.

![Bar Graph Example](image)

3.21. Gas consumption (town and country) per 100 km for four cars of different companies is shown in the bar graph. Assuming that the annual average of kilometers driven per year in the country two times more than in the city, determine which car is more economical.
3.22. The bar graph below shows energy consumption over the last two years. The solid line determines the mean monthly outside temperature. Explain a relationship between temperature and power consumption. Calculate the approximate values of the temperatures T1 and T2 averaged over season #1 (November, January, and March) as well as season #2 (May, July, and September). Also calculate the corresponding average values of electricity power consumption G1 and G2 over seasons #1 and #2, respectively.

3.23. The bar graph below shows the number of students as a percentage of the total number of younger Canadian school pupils
whose knowledge of mathematics assessed above baseline. What is the main conclusion that can be made from this graph?

3.24. 130 students took a quiz that included 5 math problems. 10 pupils solved all problems, 30 students solved 4 problems, 50 schoolchildren gave a right answer for 3 questions, 2 problems were solved correctly by 25 people, and 10 students gave a right answer for one question. Based on these data, create a bar chart that characterizes how many students did not solve one, two, three, four, and five problems.

3.25. 130 students took a quiz that includes 5 math problems. 10 pupils solved all problems, 30 students solved 4 problems, 50 schoolchildren gave a right answer for 3 questions, 2 problems were solved correctly by 25 people, and 10 students gave a right answer for one question. Based on these data, create a bar chart that characterizes how many students did not solve at least one, two, three, four, and five problems.

3.26. The chart below shows the income (arbitrary units) in four different stores. Estimate which income is more: the total for four months in the store with the highest total revenue, or total in all the stores in one month where the total revenue is the greatest.
3.27. The unemployment rate in several key provinces in Canada is shown in the diagram below. Determine the unemployment rate $X$ in Ontario if the average unemployment rate for all provinces is equal to 6.7857.

3.28. The number of flyers distributed from Monday to Sunday among homeowners is shown below, excluding Friday and Sunday. The total number of delivered flyers is 207. It is known that 3 more flyers
were distributed on Friday than on Sunday. Determine the number of flyers distributed on Friday and Sunday.

3.29. The number of flyers distributed from Monday to Sunday among homeowners is shown in below, excluding Friday and Sunday. The total number of delivered flyers is 209. It is known that 1.3 times more flyers were distributed on Friday than on Sunday. Determine the number of flyers distributed on Friday and Sunday.

3.30. The figure below shows two pie charts that define the power of the signal received by the car radio antenna. The charts determine the received signal, depending on the car position relative to cardinal directions. Explain which of the two diagrams is more preferable.
3.31. An antenna receives radio signals from different directions. The following table shows the signal levels (in arbitrary units) taken from eight different directions. Which of the radial diagrams shown below correctly reflects the data in the table?

<table>
<thead>
<tr>
<th></th>
<th>0°</th>
<th>45°</th>
<th>90°</th>
<th>135°</th>
<th>180°</th>
<th>225°</th>
<th>270°</th>
<th>315°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal</td>
<td>1</td>
<td>1</td>
<td>0.8</td>
<td>0.5</td>
<td>0.4</td>
<td>0.7</td>
<td>1.2</td>
<td>1.3</td>
</tr>
</tbody>
</table>

![Radial diagrams](image)

1. ![Diagram 1](image)
2. ![Diagram 2](image)
3. ![Diagram 3](image)
4. ![Diagram 4](image)
3.32. Radio antennas of two identical cars receive signals from the same transmitting tower. Signal power, depending on the car angle orientation, is shown in the figure. Find the angle $\phi$ between the cars.

3.33. The diagram below schematically presents the signal power received by a car’s FM radio antenna as a function of an angle between the incoming signal direction and the orientation of the car as shown in the diagram. The car is located in the north-south direction as shown in the figure. Data are presented for two different models. Specify two features that characterize the received signals for different cars.
3.34. The diagram below shows the probability that a family has one, two, or three kids. The horizontal axis shows the number of children in the family, and the vertical shows the corresponding probability. On the basis of the chart, estimate an average number $\bar{N}$ of children in the family.

![Bar Graph](image)

3.35. The diagram below shows the probability that a family has one, two, or three kids. The horizontal axis shows the number of children in the family, and the vertical shows the corresponding probability. On the basis of this chart, create a chart that reflects the probability that family has at least one, two, or three kids.

![Bar Graph](image)

3.36. A math teacher summarized the results of the control work. The results are presented in a pie chart. Sector A (12%) corresponds to
the number of students with the highest score Excellent. Sector B (15%) corresponds to the number of pupils that have good results. Sector C (56%) corresponds to the number of students rated with score Pass. Finally, sector D (17%) corresponds with the number of students who did not take the test.

![Pie Chart](image)

Which of the following statements regarding the test results is wrong?

a. More than half of the students received a score Pass.
b. More than a quarter of students did not take the test.
c. Less than 25 percent of students received a highest score.
d. The number of students who received a score Pass, Good, or Excellent is more than half of the total number of pupils that took the test.

3.37. The number of student from grades five, six, seven, and eight who attend advanced math class is shown in the figure below.
Which statement is true regarding the students in advanced classes if the total amount of students is 360?

a. There are no advanced students in grade five.
b. The number of students in advanced grade eight is more than the number of students in advanced grade seven.
c. More than half of the total amount of students is not from grade seven.
d. The number of students from grade six is less than eighty-eight.

3.38. 100 athletes from four countries participated in international competitions. The figures below show the distribution of athletes from different countries and the ratio of the winners to the total number of participants. Medalists came in second or third place. Participants are athletes who are not winners or medalists.
Which statement is true?

a. At least five winners are not from the United States.
b. All athletes from the United States became winners.
c. At least one athlete from China was a winner.
d. At least five Russian athletes were winners.

3.39. The store sells toys of different sizes: large, medium, and small. The figure below shows the relationship between the number of different-sized toys. Determine the number of toys of each size if the total number of toys is 500.

3.40. The store sells toys of different sizes: large, medium, and small. Figure 1 shows the relationship between the number of different-sized toys. Figure 2 shows the number of different-colored toys.
Bar and Pie Graphs (Problems)

Figure 2

Which one of the following statements is true?

a. There are no small, yellow toys.
b. Some blue or red toys are medium sized.
c. More than half of the total number of toys is blue and yellow.
d. Green toys are not large.

3.41. There are one hundred employees in a company. Each employee knows at least one foreign language (English, French, or German). The following chart reflects the number of people who knows each language.

The pie graph below reflects information about employees who know one, two, or three languages.
Determine the number of people who speak only English.

3.42. There are one hundred employees in a company. Each of employees knows at least one foreign language (English, French, or German). The following chart reflects the number of people who know each language.

The pie graph reflects information about employees who know one, two, or three languages.
Determine the number of people who speak only French and English if two people know English and German but don’t know French.

3.43. Figure 1 shows the relationship between the different-sized toys offered. Figure 2 demonstrates their color distribution.

The total amount of toys is 200, green and blue toys are only medium sized, and red and yellow toys are not medium sized. Determine the number of green and yellow toys in figure 2.

3.44. The number of cars and their colors in the underground parking lot are shown in the bar graph below. Determine the number of white and silver cars if the total number of cars is 520 and the ratio of red cars to white cars is equal to the ratio of black cars to silver cars.
3.45. The number of cars and their colors in the underground parking lot are shown in the bar graph below. Determine the number of white and silver cars if the total number of cars equals 520 and the difference between the number of white and silver cars is equal to 60.

3.46. The number of cars and their colors in the underground parking lot are shown in the bar graph below. Determine the number of white and silver cars if the difference between the total number of vehicles and the total number of red, white, and brown cars is two hundred. The number of black and silver cars is equal to the number of red and white cars.
3.47. A shop sells shirts, sweaters, and hats of different sizes: A, B, C, and D. Figure 1 shows an item’s size distribution. Figure 2 highlights the number of different goods.

Figure 1

Figure 2
Which of the following statements is certainly not true?

a. Goods of size D account for 25 percent of the total quantity of goods.
b. All shirts and hats can be of the same size.
c. All sweaters and hats may not be of the same size.
d. The number of sweaters is more than the number of the size-A goods.

3.48. A shop sells shirts, sweaters, and hats of different sizes: A, B, C, and D. Figure 1 shows an item’s size distribution. Figure 2 highlights the number of different goods.
Bar and Pie Graphs (Problems)

Which of the following statements may be true?

a. All shirts can have the same size.
b. All hats and sweaters are of the same size.
c. All sweaters can be size A or B.

3.49. The figure below shows a four-way stop with cars #1, #2, #3, and #4 arriving at the intersection. A motorist approaching an all-way stop is always required to come to a full stop (about three seconds) before the crosswalk. After a full stop has been made, vehicles usually have the right-of-way to proceed through the intersection in the order that they arrived at the intersection. Assuming that each car leaves the intersection 3 seconds after the previous one, estimate the time delay between the moment when the first car (in lane 1) arrived to the intersection and last car (in lane 4) left the intersection.

3.50. The figure below shows the four-way stop with the arrived cars. A motorist approaching an all-way stop is always required to come to a full stop before the crosswalk. After a full stop stop (about three seconds), vehicles usually have the right-of-way to proceed through the intersection in the order that they arrived at the intersection. According to the scenario presented in the figure below, estimate the time delay between the moment when the first car (in lane 1) arrived to the intersection and last car (in lane 4) left the intersection.
Bar and Pie Graphs (Problems)
Chapter 4

Logic

Problems

4.1. One corner of a rectangular table is sawed off. How many angles
does the table have now?

4.2. If a kangaroo jumps 1.5 times farther than he is now, he’ll need
exactly 6 jumps to get the tree. How many jumps will the kangaroo
need now?

4.3. What sign do you need to put between digits 4 and 5 to get a
result that is more than 4 and less than 5?

4.4. Five fishermen eat five pike in five days. How many days will it
take for ten fishermen to eat ten pike?
4.5. Warehouse workers have to relocate six boxes of goods from one place to another. Working in pairs, each pair of workers moves three boxes. How many workers did this job?

4.6. It is rainy in the city at midnight. Can we hope that after 96 hours, the sun will come out?

4.7. The hour hand of a clock turns 45 degrees starting from 12 am. Determine what time is it now.

4.8. How many times per day do the hour hand and minute hand on a clock form a right angle?

4.9. Could you make a straight line equal to 100 cm using lines of 7 cm and 12 cm?

4.10. There are 15 chairs in the room. Propose how to arrange chairs so that they form three rows, with six chairs in each row.
4.11. A dad and two sons went camping. On the way, they encountered a river and a raft on the shore. The raft can hold only the dad or the two sons on the water. How do they cross over the river to get to the other side?

4.12. A cylindrical water container is full. Without using any measurement instruments, draw a picture to show how to empty exactly half of the water volume.

4.13. How can exactly 4 liters of water be measured if you have empty bottles of 3 and 5 liter capacity? You can pour water from one bottle to another, pour water into the bottles, or you can empty bottles. Do not use any measurement container.
4.14. You have two empty cylindrical pails. One has a capacity of 6 liters, while the other can hold exactly 11 liters. Your task is to go to a pond and come back with exactly 8 liters of water. How do you do it?

4.15. A container with 8 liters of water in it is full. You need to leave 4 liters in the container, and you have empty bottles of 3 and 5 liter capacity. How do you do it?

4.16. You have three containers of 19, 13, and 7 liters. The 13 and 7 liter containers are full of water, while the 19 liter container is empty (20 liters of water in total). Could you pour water between the containers to fill two of them with 10 liters of water?

4.17. Daniel has a 6 liter container full of water, and empty bottles 5 and 1 liter capacity. Without any measuring instruments, how can Daniel divide the water in such a way that the 5 and 6 liter containers will contain 3 liters of water (each)?

4.18. Find a solution to the following:

\[
\begin{array}{c}
+ \\
** \\
* \\
**8 \\
\end{array}
\]

70
4.19. Find a solution to the following:

\[ \begin{array}{c}
+ ** \\
** \\
*98 \\
\end{array} \]

4.20. Insert the missing digits into the following:

\[ \begin{array}{c}
* 93 \\
+ ** \\
** 51 \\
\end{array} \]

4.21. Olga, Tanya, Yelena, and Irina have prepared jam sandwiches. Two girls used currant jam, while the two others used gooseberry jam. Tanya and Irina did not use the same kind of jam, and Irina and Olga also used different jams. Irina has a sandwich with gooseberry jam. Which berry did each girl use to make her sandwich?

4.22. Anna, Vicky, Gail, and Nadia are standing in a circle. The girl in the green dress (not Anna and not Vicky) is located between the girl in the blue dress and Nadia. The girl in the white dress is located between the girl in the pink dress and Vicky. What color dress is each girl wearing?
4.23. Two identical cups are filled with the same quantity (100 grams) of milk and coffee, respective. With a spoon, one takes 10 grams of milk from the first cup, pours it into the cup of coffee, and mixes it with the spoon. Then, with the same spoon, one takes exactly the same quantity as before from the cup containing the mixture of coffee and milk and pours it into the cup of milk, then mixes it. How many grams of coffee were poured into the cup with milk? Answer in fraction form.

4.24. Two identical cups are filled with the same quantity (100 grams) of milk and coffee, respectively. With a spoon, one takes 10 grams of milk from the first cup, pours it into the cup of coffee, and mixes it with the spoon. Then, with the same spoon, one takes exactly the same quantity as before from the cup containing the mixture of coffee and milk and pours it into the cup of milk, then mixes it. Which quantity is greater—the coffee in the cup of milk, or the milk in the cup of coffee?

4.25. A boy carries one kilogram of iron in one hand and the same amount of fluff in the other hand. What is heavier?

4.26. A cup of coffee with a spoonful of sugar costs $1.10. It is known that coffee is more expensive than a spoon of sugar by $1. How much is the coffee itself and how much is the spoonful of sugar?
4.27. There are five apples in a basket. How do you equally divide five apples among five men and to leave one apple in the basket?

4.28. A warehouse worker can lift a package of $3 \times 3 \times 3$ liter milk packs. Would three workers be able to lift a package of $9 \times 9 \times 9$ liter milk packs?

4.29. There is a basket with candies on the table. Some kids took half of the total amount of candy from the basket, then another half of the amount that remained, then half of the newly remaining part, and finally half of the next remainder. After this, there were 10 candies in the basket. How many candies were originally in the basket?

4.30. With each stroke, an athlete splits a piece of concrete into three parts. How many pieces has he split if the concrete lump is in 2007 parts?
4.31. There are 15 black socks and 20 white socks in a closed drawer. What is the minimum number of socks you have to pull out in the dark to get at least one pair of the same color?

4.32. The first seller has a 15 kg bag of flour and the second seller has a bag weighing 20 kg. Each of them divides his own bag into a few small bags weighing 2 or 3 kg. Is it possible to divide both sacks such a way that the first and second sellers would have the same number of small bags? How many bags will each seller have?

4.33. There are 12 students who speak English in the class, 7 who Chinese, and 5 who know both languages. How many people are there in the class?
4.34. One teabag can make 2 or 3 cups of tea. Mila and Tanya divided a box of teabags equally. Mila brewed 23 cups of tea, and Tanya, 33 cups. How many bags were in the box?

4.35. There is a pile of 80 coins, all of equal size. One of the coins is false, and it is lighter than others. What is the minimum number of coins that must be weighed on the balance scales to identify the false one?

4.36. Let’s assume that you have 101 coins that are identical in form. Among them, one is fake, and this is characterized by a different weight. Using balance weighing scales without standard masses of known weight, how can you determine whether the fake coin is lighter or heavier than a normal coin? It is not necessary to find the counterfeit coin.

4.37. A teacher announced some test results: Serge did not receive an A, Victor did not receive a B, and Alex did receive a B. It is known that the teacher made two mistakes. Figure out which student received A, who had a B, and who had a C if it is known that one of them received a C, one received a B, and one received an A.

4.38. A family has four children of 5, 8, 13, and 15 years old. Their names are Anna, Boris, Vera, and Gail. How old is each child if a girl goes to kindergarten, Anna is older than Boris, and the ages between Anna and Vera added together make a number divisible by 3?
4.39. Thirty-five students wrote a math test. They received 120 sheets of paper in total, each boy received one sheet more than a girl. How many boys and girls wrote the test, if all the girls were given the same number of sheets as all the boys?

4.40. Maria gets up at 8 o’clock in the morning. She knows that her alarm clock is slow, and the delay is equal to 8 minutes for 4 full days. At what time does the alarm clock go off on Wednesday morning if set it to the correct time at 8 pm the previous Saturday?

4.41. Four friends—Victor, Daniel, Shawn, and Phillip—were fishing. Victor caught more fish than Daniel, while Shawn and Phillip together caught as many as Victor and Daniel. Victor and Shawn caught less than Daniel and Phillip. Who caught the most fish and who caught the least?
4.42. The number of boy students that are sitting at a desk with other boys is two times more than the number of girls sitting with the girl students. The number of girls sitting with boys is equal to the number of boys that are sitting with boys. What is the ratio of the number of boys to the number of girls if it is known that only two students sit at one desk?

![Image of students sitting at a desk]

4.43. Over five years, a student passes 31 exams. Each year, he writes more exams than in the previous year, and at the end of the fifth year, he passes three times more exams than he did at the end of the first year. How many exams did he pass at the end of the fourth year?

4.44. In one month, three Wednesday days fall on even dates. What date number will the second Sunday of the current month fall on?

4.45. Let’s assume that you have five suitcases and five keys to them, but you do not know which key fits which suitcase. How many attempts will be necessary in the worst case to find the appropriate key for each suitcase?

4.46. There are 37 students in a group. Is there any month in the year in which at least four students in the group celebrate their birthdays?
4.47. A box contains blue, red, and green pencils. The total number of pencils is 20. The number of blue pencils is six times greater than that of green pencils, and the number of red pencils is less than the number of blue pencils. How many red pencils are in the box?

4.48. The middle of three brothers is older than younger one by two years, and the older brother is 4 years older than the total age of the youngest and middle brothers. Find the age of each brother, when the sum of their ages is 96 years.

4.49. Leon and Irina live in a building where each floor has nine apartments (there is one entrance to the building). The number of the floor on which Leon lives is equal to the number of Irina’s apartment. The sum of Irina and Leon’s apartment numbers is 329. What is Leon’s apartment number?

4.50. A boy comes back from the zoo, and tells his sister: “I saw tigers and monkeys. The number of tigers was seven times greater than that of the other animals, and the numbers of monkeys was seven times less than that of the other animals. However, I don’t remember how many lions I saw.” Can you tell how many lions the boy saw?
4.51. Each member of a team shot at a target 10 times at a shooting competition. Five points are awarded for every successful result and 2 points are lost if the shooter does not hit the target. A winner must scores at least 30 points. How many times does a member of team have to hit the target to be among the winners?

4.52. A student reads a book. When reading with a speed of $V_1=1$ page per minute, he finishes it 2 hours earlier than with a speed of $V_2=0.5$ pages per minute. How many pages are in the book?

4.53. Three containers have 240 liters of waters. Forty-eight liters of water were poured from the first tank and 72 liters were poured from a second tank. Therefore, each of these tanks contains three times less water than a third tank. How many liters of water were in each tank initially?

4.54. The number of cents that are in the pockets of each of the three friends increases as an arithmetic progression from one guy to the other. If we subtract the digit 2 from the second term of this progression and leave the remaining numbers unchanged, it will become a geometric progression. Find the number of cents in each friend’s pockets if their sum is equal to 30.

4.55. Vladimir has two more sisters than he does brothers. How many more daughters than sons do Vladimir parents have?

4.56. There are five heads and fourteen feet in the family. How many
family members are people, and how many are dogs?

4.57. There are 23 students in the group. Victor is taller than 17 students and 13 students are shorter than Peter. How many people are taller than Victor and shorter than Peter?

4.58. A ferry captain can load 10 cars or 6 trucks on the ferryboat. On Thursday, the fully loaded ferryboat crossed the river five times and transported 42 vehicles. How many of them were trucks?

4.59. Workers have a special truck with a 3000 kg capacity to transport containers with a weight of 170 and 190 kg. Is it possible to fully load the truck to transport these containers?

4.60. Three boys of different ages—Alex, Boris, and Cohen—are playing tennis. It is known that one of the following statements is false:

a. Alex is older than Boris;
b. Cohen is younger than Boris;
c. Cohen is older than Alex;
d. The sum of Boris and Cohen’s ages is twice Alex’s age.

Who is the youngest boy?

4.61. Two parliament chambers have an equal number of deputies. All members of both chambers have voted on an important issue with no abstentions. When the chairman says that the decision has been made by a margin of 23 votes, the leader of the opposition says that the voting was rigged. Is this true?
4.62. Yuri, Michael, Vladimir, Scott, and Otto wait in the line to buy a movie ticket. It is known that:

a. Yuri is before Michael in line, but after Otto;
b. Vladimir and Otto aren’t standing near each other;
c. Scott is not near Otto, Yuri, or Vladimir.
Who is first in line, and who is second, third, fourth, and fifth?

4.63. There are 70 kids at a scout’s camp. Twenty-seven children are in the drama club, 32 sing in the choir, and 22 teen-agers are sportsmen. It is known that 10 kids from the choir are in the drama club, 6 sportsmen are in the choir, and 8 sportsmen are in the drama club. In addition, 3 sportsmen attend the drama club and choir. How many kids don’t sing, are not sportsmen, and don’t attend the drama club?
Chapter 5
Linear and Circular Motion

Basic Formulas

Distance/speed/time word problems, also called “uniform rate” problems, involve an object traveling at some fixed and steady (uniform) pace (rate or speed) or moving at some average speed. Whenever you read a problem that involves the phrases “how fast,” “how far, or “for how long,” you should think of the distance equation:

a. Linear (straightforward) motion

\[ S = V \times T \]

S = distance, V = speed, T = time
b. The average speed $V_{av}$ is always total length divided by total time. If half of the distance object travels at a speed of $V_1$ and the other half of the distance object travels at a speed of $V_2$, then average speed can be calculated as

$$V_{av} = \frac{\text{Total Distance}}{\text{Total Time}} = \frac{S}{\frac{S}{2V_1} + \frac{S}{2V_2}} = \frac{2V_1V_2}{V_1 + V_2}$$

c. Relative motion

Velocity of the moving objects with respect to other moving or stationary objects is called *relative velocity*, and this motion is called *relative motion*. Reference point is very important.

**Example 1.** According to observer A, observer B is traveling at a speed of 30 km/h to the east. However, according to the observer in B, observer A is traveling with 30 km/h to the west.

![Diagram](image)

**Example 2.** Let’s say two objects move as shown below.
Calculate the velocity $V$ of observer B with respect to observer A.

The answer is shown below with the Pythagorean theorem, $V = \sqrt{V_1^2 + V_2^2}$:

$d$. Circular motion

Uniform circular motion describes the motion of an object traversing a circular path at constant speed $V$. For motion in a circle of radius $R$, the circumference of the circle is $S = 2\pi R$. If the period for one rotation is $T_{rot}$, the angular rate of rotation, also known as angular velocity, $\omega$ is:
Linear and Circular Motion (Problems)

\[ \omega = \frac{2\pi}{T_{\text{rot}}} \] (the units are radians/second)

The speed of the object traveling along the circle is:

\[ V = \frac{2\pi R}{T_{\text{rot}}} \]

The angle \( \theta \) swept out in a time \( t \) is:

\[ \theta = 2\pi \frac{t}{T_{\text{rot}}} \]
Problems

5.1. How long will it take a car to travel between two cities that are 120 km apart, when the car is traveling at a speed of 100 km/h?

5.2. At what speed must the car be traveling if it takes one and a half hours to travel between two cities that are 120 km apart?

5.3. A train of one kilometer in length travels at a rate of 60 km/h. How long will it take to pass through a one-kilometer-long tunnel?

5.4. A car travels a distance of S=600 km in T hours with a speed equal to V = 90 km/h. If the speed increases by 20%, then the moving time becomes T-1. Find the value of T.
5.5. The first half of a trip, a freight train travels at a speed of $V$ km/h. Then, automatic railway signaling system stops the train for 2 hours. The train then proceeds at a speed of $(V + 10)$ km/h and arrives at the destination without a delay. Find the distance $S$ if without stoppages the train at a speed of $(V - 10)$ km/h covers all way 6 hours later with respect to a trip at a speed of $V$.

5.6. The first half of a trip, a freight train travels at a speed of $V$ km/h. Then, automatic railway signaling system stops the train for two hours. The train then proceeds at a speed of $(V + 10)$ km/h and arrives to the destination without a delay. Find the speed $V$ if without stoppages train at a speed $(V - 10)$ km/h covers all the way by 6 hours later compared with a trip at a speed of $V$.

5.7. A traveler walked 10 km and stopped for lunch. After a half an hour, he continued to move 1 km/h faster and reached the destination without delay. Find the initial speed of the traveler if the first part of the way before lunch is equal to 50% of the entire route.
5.8. An airplane makes a round trip from point A to point B and back. During the flight, there is a constant wind speed of 25 km/h from point A to point B. What is the ratio between the times it takes the airplane to travel one way versus the time it takes to fly back to point A? The speed of the airplane without any wind is 625 km/h. Round the answer to the nearest tenth.

5.9. Cities A and B are located along a river and are S km apart. A boat travels downstream from A to B in time $t_1$ and back from B to A upstream in time $t_2$. The total time is $T_1 = t_1 + t_2$. If a boat travels from A to B and back from B to A in still water, the total time is $T_2$. What time is less $T_1$ or $T_2$?
5.10. Tourists have to travel 25 km in four days. The second day, they covered 10 km more than the first day. The third day, they traveled 5 km less than the first day. The fourth day, they covered the same distance as in the first day. How many kilometers did they travel every day?

5.11. A boatman running river upstream lost a bottle when he was under a bridge. He noticed the loss 15 min later, turned back, and discovered the bottle 2 km from the bridge. Find the water’s rate of speed.

5.12. A woman walked down a downward-moving escalator and counted $n_1 = 100$ steps to reach the bottom. Then, she ran back up
the downward-moving escalator at the same speed and took 300 steps to reach the top. How many steps \( n \) would she cover on the escalator when it is switched off (does not move)?

5.13. A man who is two meters tall stands 4 meters from a lamppost that is 10 m high. How far away from the man is the tip of his shadow (point D)?

5.14. A two-meter-tall man stands 4 m from a ten-meter tall light post as shown in the attached figure below. He has walked 6 m away from his initial position. What is the distance between the new and the initial shadow tip positions (tip position is determined by point D in the figure)?
5.15. An h-meter-tall man stands $L_0$ meters from an H meter tall light post (H$h$). He starts to walk away from the light pole at a speed of $V_1$. The shadow tip (point D) walks at a speed $V_2$ with respect to the light pole. Find the ratio $q = \frac{V_2}{V_1}$ as a function of H and h values.

5.16. An h-meter-tall man stands $L_0$ meters from an H meter light pole. He starts to walk away from the pole at a speed of $V_1$, as shown in the figure below. His shadow drifts at a rate of $V_2$ with respect to a man. Estimate a ratio $q = \frac{V_2}{V_1}$ as a function of H and h.
5.17. The distance between two cities is 240 km. The first half of the trip, the vehicle travels at 100 km/h. The remainder of the trip, the vehicle travels at 80 km/h. What is the average speed of the vehicle?

5.18. The distance between two cities is 240 km. The first half of the trip, the vehicle travels at 100 km/h and then rests for 30 min at a gas station. The remainder of the trip, the vehicle travels at 80 km/h. What is the average speed of the vehicle?

5.19. The first third of a trip, a vehicle travels at 100 km/h. It then rests for the equivalent of a third of the time that it has driven so far. The remaining two-thirds of the trip, the vehicle travels at 120 km/h. Determine the average speed of the vehicle.

5.20. A driver knows that to get to Pearson Airport on time from Toronto, he has to drive at a rate more than $V_0 = 50$ km/h. However, the first half of the route was very busy, and he traveled at a speed of $V_1 = 30$ km/h. The second part of the route, he traveled with a speed $V_2 = 100$ km/h. Does he get to the airport on time?

5.21. Cities A and B are located along a river and are $S$ km apart. The rate of still water is $V_1$, and the speed of a boat in still water is $V_0$. Find an average rate for a trip along the river between cities A and B if the boat travels from A to B and back from B to A.
5.22. Cars travel in same direction in three parallel lanes along highway. The inner lane stream moves at a speed of 90 km/h, the center stream moves at a speed of 105 km/h, and the outer stream rides at a rate 120 km/h. Find the average rate of speed of a car if it travels 40 min in the inner lane, 20 min in the central lane, and one hour in the outer lane.

5.23. A bus left City A toward City C. A car left City A half an hour after a bus (also headed toward City C) and overtook a bus at a distance of 50 km from City A. Find the speed of a bus if it travels 50 km/h slower than the speed of the car.

5.24. A bus left City A driving at 60 km/h toward City C. A car left City A half an hour after a bus (also headed toward City C) driving at 80 km/h. How long would it take the car to overtake the bus if the distance between City A and City C is 90 km?

5.25. Cyclist left City A toward City B at the same time a driver left City B toward City A. They met each other. The cyclist arrives in City B four hours after meeting while the car arrives in City A one hour after meeting. Find the cyclist’s total moving time from City A to City B.

5.26. Two trains, traveling toward each other, left from two stations that are 192 km apart. The first one left City A for City B at 7:00 a.m. The second one left City B for City A at 8:00 a.m. If the rate of the first train is 60 km/h and the rate of the second train is 28 km/h, how far from City A will the two trains pass each other?

5.27. Two trains started from the same location, traveling in opposite directions. The first train left at 6:00 a.m. at a rate 15 km/h slower than the second train, which left at 6:30 a.m. They were 135 km apart at 7:00 a.m. Find the rate of speed of the second train.

5.28. Two cyclists start at the same time from opposite ends of a course that is 140 km long. One cyclist is riding at 15 km/h, and the
second cyclist is riding at 20 km/h. How long after they begin will they meet?

5.29. A car and a bus set out at 2:00 p.m. from the same point, headed in the same direction. The average speed of the car is 30 km/h slower than twice the speed of the bus. In two hours, the car is 20 km ahead of the bus. Find the rate of the car.

5.30. Sprinters A and B run a loop around a closed-loop track from the same point. Runner B starts two seconds later than runner A. When will they meet again if a fast runner completes one loop in $T_1 = 65$ seconds and a slow runner covers one loop in $T_2 = 70$ seconds?

5.31. The picture shows cars moving in different direction at an intersection. The traffic density of cars moving in direction 1 is equal to 25 cars a minute, and the traffic density of cars moving in direction 2 is 80 cars a minute. The traffic light for cars traveling in direction 1 is green for two minutes straight until it turns yellow. How long must the green light be on for cars traveling in direction 2?

5.32. The passenger of a train moving at 60 km/h noticed that an oncoming freight train traveling in opposite direction at a speed of 30 km/h passed him in 10 sec. What is the length of the freight train?
5.33. The passenger of a train moving at a speed of 60 km/h noticed that a freight train that is traveling in the same direction at a speed of 30 km/h, passed him in 10 sec. What is the length of the freight train?

5.34. The passenger of a passenger train noticed that an oncoming freight train traveling at a speed of $V_1 = 30$ km/h with a length of 500 meters in the same direction passed a passenger train in 30 seconds. What is the speed of the passenger train?

5.35. Passenger and freight trains start at the same time from opposite ends of a course that is 1 km long. The passenger train is traveling at 90 km/h, and the freight train is traveling at 60 km/h. How long after they begin will they meet?

5.36. Two cars leave a city at the same time with the same speed. One car is traveling west at an average speed of 90 km/h, while the other train is traveling east an average speed of 120 km/h. In how many hours will they be 1,260 kilometers apart?

5.37. A passenger on a train traveling at 115 km/h walks toward the back of the train at a rate of 5 km/h. What is the passenger’s rate of travel with respect to the ground?
5.38. A passenger on a train traveling at 115 km/h walks toward the train moving at a rate of 5 km/h. What is the passenger’s rate of travel with respect to the ground?

5.39. A passenger on a train traveling at 115 km/h walks toward the back of the train at a rate of 5 km/h. What is the passenger’s rate of travel with respect to a person who walks along the railway platform toward the back of the train moving at a speed of 3 km/h?

5.40. During the first two hours of a trip, a cyclist travels north at a rate of 15 km/h. For the next four hours, he drives east at a speed of 10 km/h. For the third part of the trip, the cyclist moves southwest at a speed of 20 km/h for two and a half hours. Which graph gives the right route of a cyclist?
5.41. A man crosses a river with a width of $S_1 = 100$ meters at a rate of $V_1 = 0.25$ m/sec with respect to still water. The speed of a stream is $V_2 = 0.1$ m/sec. Find the distance $S_2$ that the man drifts along the stream.

5.42. Two cyclists left from the same point as shown in the figure below. The first cyclist headed directly east, and the second headed northeast at an angle of 60° from the first cyclist’s path. What will be the distance between the cyclists in the span of one hour if the first cyclist is moving at a speed of 20 km/h while the second is moving at a speed of 10 km/h?
5.43. A bicycle wheel has a radius of \( R = 40 \text{ cm} \). Find the linear bicycle speed \( V \) if a wheel makes one hundred rotations per minute.

5.44. A bicycle wheel is \( R \) centimeters in diameter. How far does point A go in \( N \) revolutions of the wheel?

5.45. A bicycle wheel is 0.75 meter in diameter. To the nearest revolution, how many times \( N \) will the wheel turn if the bicycle is ridden for 4.8 kilometers?

5.46. The earth rotates about its axis once every \( T = 24 \text{ hours} \). The radius of the earth along the equator is approximately \( R \approx 6400 \text{ km} \). Find the linear (meter/second) speed \( V \) of a point on the equator.

5.47. We orbit the Sun at a distance of about 150 million kilometers. Find the linear speed of the earth’s rotation around the Sun.
5.48. A cyclist completes a loop around a circular track of radius $R$. His bicycle wheel has a radius equal to $r$. How many times does the wheel of a bicycle turn if the bicyclist has made one circle?

5.49. A person has a measuring wheel that records the distance the wheel is rolled along the ground. The diameter of the wheel is 1 m. If the wheel is pushed half a kilometer, how many times will the wheel go around?

5.50. The train speed is equal to $V = 72$ km/h. How many revolutions per minute do the locomotive wheels, the radius of which is $R = 1.2$ meter?
5.51. Two cyclists traveled the same road between City A and City B. One bicycle has a wheel diameter 1.3 greater than the other one. Find the ratio of wheel revolution of the big wheel with respect to the small wheel.

5.52. Ferris wheel cars are labeled as 1, 2, 3, 4, and so on. When the car with the number 25 is at the top point of a wheel, the car with a number 8 is located at the lowest point. How many cars are on the Ferris wheel?

5.53. Shawn is on a Ferris wheel that spins at the rate of three revolutions per minute. The wheel has a radius of fourteen meters, and the center of the wheels is eighteen meters above the ground. After the wheel starts moving, it takes Shawn 5 sec to reach the top. How high above the ground is he when the wheel has been moving for 45 sec?

5.54. A wall clock’s second hand has a length equal to \( L = 10 \) cm. Find a linear speed of a hand tip.

5.55. Calculate the angular velocity of the minute hand of a clock.
5.56. What is the length of the minute hand if its tip gains 1.25 cm in one minute?

5.57. Lifting water from the well shaft takes twenty revolutions. What is the depth of the well in meters if the shaft diameter is $D = 0.2$ m (Let $\pi \approx 3$)?

5.58. Two cyclists are moving from point A to point B. The first moves along a straight line AB, and the second moves along a semicircle with a diameter length AB. What should be the relationship between the speeds of cyclists if they come from A to B at the same time?

5.59. How many revolutions per minute does the gear with 32 teeth make if an attached gear with eight teeth makes 12 revolutions per minute?
5.60. The gear diameter ratio as shown in the figure is equal to 3:8. At what angle will the big wheel turn when the small one completes one revolution?

5.61. Gears #1, #2, #3, #4, #5 have 31, 19, 13, 9, and 5, teeth, respectively. Gear #4 makes one revolution. Find angles that gears #1, #2, #3, and #5 turn on.

5.62. A sled travels down a hill, passing each subsequent second 5 m more than the previous one. How long will it take to get the bottom of the hill if it travels 2.5 m in the first second and the length of the hill is equal to 1,000 meters?

5.63. Victor visited the Grand Canyon and dropped a penny off the edge of a cliff. The distance the penny will fall is 5 m the first second, 15 m the next second, 25 m the third second, and so on in an arithmetic sequence. What is the total distance the penny will fall in 6 sec?

5.64. A ball on a thin string rotates uniformly in an XY plane as shown in the figure below. The string breaks when the angle between the string and the horizontal axis is 45°. How long does it take for the ball to cross the x-axis if the string length is one meter and the speed of the ball is 0.1 m/sec?
5.65. A thread is wound on a coil reel with automatic winding. Determine how long it will take the coil to make 30 turns if the number of turns varies with time in accordance with the law \( N = \alpha \times t + \beta \times t^2 \), where \( \alpha = 2 \) turns/second and \( \beta = 0.1 \) turn/second².

5.66. A thread is wound on a coil reel with automatic winding. Determine how long it will take the coil to make 32 turns if the number of turns varies with time in accordance with the law \( \varphi = \alpha \times t \), where \( \alpha = 4 \) turns/second.

5.67. A ball is thrown upward. The height of the ball varies according to the law \( h = 1.6 + 8t - 5t^2 \) where \( h = \) height in meters and \( t = \) time in seconds. How many seconds will it take for the ball to be at a height of more than 3 meters?

5.68. A ball is thrown upward. The height of the ball varies according to the law \( h = 1.6 + 8t - 5t^2 \) where \( h = \) height in meters and \( t = \) time in seconds. How many seconds will it take for the ball to be at a height of more than 1.6 meters?

5.69. A person standing on a balcony throws a ball straight up. The height of the ball above the ground varies according to the law \( h = 25 + 20t - 5t^2 \) where \( h = \) height in meters and \( t = \) time in seconds. After how many seconds \( t \) will the ball hit the ground?
5.70. A person standing on a balcony throws a ball straight down. The height of the ball above the ground varies according to the law 
\[ h = 25 - 20t - 5t^2 \] where \( h \) = height in meters and \( t \) = time in seconds. After how many seconds \( t \) will the ball hit the ground?
Linear and Circular Motion (Problems)
Chapter 6
Math for Physics

Problems

I. A law of conservation of mechanical energy in motion

The mechanical energy of an object is the sum of potential energy and kinetic energy. This is associated with the motion and position of an object. The principle of conservation of mechanical energy states that in an isolated system, the mechanical energy is constant. The total energy of the moving object is \( E_{total} = E_p + E_c \), where \( E_p = mgh \) is the potential energy of the object at the height \( h \) above the ground, \( m \)=the mass of the object, and \( g \)=gravity acceleration. The kinetic energy of an object that moves with the speed \( V \) is equal to \( E_k = \frac{mV^2}{2} \). When the object gets closer to the ground, for example, its potential energy decreases, while its kinetic energy increases.

6.1. Using the law of conservation of energy, determine that velocity of a ball when it reaches earth if it falls from a height equal to \( h \) above the ground.

Hint: When the moving ball stays in motion the, total mechanical energy \( E_{total} = E_p + E_c \), where \( E_p = mgh \) is the potential energy of the ball at a height of \( h \) above the ground and \( E_k = \frac{mV^2}{2} \) is the kinetic energy of the ball moving at the speed of \( V \).
6.2. Evaluate the velocity of a wheel rolling along a track from a level h above the ground (the wheel does not slip as it rotates).

6.3. Using the law of conservation of energy, evaluate the velocity of a ball at a height of h/2 if it is falling from a height of h above the ground.

6.4. A particle is traveling without friction up and down hills and valleys. The initial height of the particle above the ground is h and the initial kinetic energy is three times less than the initial potential energy. Evaluate the speed V of the particle at a level of h/4 above the ground. Ignore friction between the particle and the ground.

6.5. Suppose you drop a tennis ball from a height of h=2 m with a speed of V. After the ball hits the floor, it rebounds to a height of H=4 m. Evaluate the initial speed V. Let g=10 m/sec².
6.6. A cannonball is fired horizontally from the top of a cliff at a height $H$ above ground level. An initial kinetic energy is three times less than the potential energy at the height $H$. Evaluate the angle between the horizon and the ball velocity direction $V$ when the cannonball strikes the earth.

Hint: The force of gravity acts only in a vertical direction; therefore, it is convenient to separate the velocity of the ball at ground level into two perpendicular components, as shown in the attached figure. The horizontal velocity component ($V_x$) describes the influence of the velocity in displacing the projectile horizontally. The vertical velocity component ($V_y$) describes the influence of the velocity in displacing the projectile vertically. Perpendicular components of motion are independent of each other.

6.7. A ball is propelled from the ground at an angle of $45^0$ to the horizon with an initial speed of 20 m/sec. Evaluate the maximum height $H$ of the ball above the ground if the initial height is $h=1.6$ m. Assume that $g=10$ m/sec$^2$.
6.8. A ball is propelled from the ground at an angle of $45^0$ to the horizon with an initial speed of $V_0=20$ m/sec. Evaluate the horizontal range $D$ reached by the ball if the time of the ball’s motion from the earth level to the maximum height is determined by the formula $t = \frac{V_y}{g}$ and the initial height level is 0.

6.9. Metal balls whose masses are $m_1=100$ g and $m_2=300$ g are suspended by equally light strings with equal lengths of $l=50$ cm, such that the balls touch when in their equilibrium position. We pull the first ball back until its string makes an angle $\theta=90^0$ with the vertical and let it go. It collides elastically with the second ball. How high will the second ball rise above its starting point? Blow perfectly elastic. Air resistance is not considered.

Hint: Take into account the following formula: $m_1V_1 = m_2V_2 + m_1U$. Here, $V_1$ is the speed of the first ball when the string of the first ball is vertical (before collision with the second ball), $V_2$ is the initial speed of the second ball when the string of this ball is vertical (after the collision with the first ball), $U$ is the speed of the first ball just after collision with the second ball when the string of this ball is vertical. An elastic collision is an encounter between two bodies in which the total kinetic energy of the two bodies after the encounter is equal to their total kinetic energy before the encounter.
II. Liquids and Archimedes’ principle

If an object is submerged in a liquid, the object displaces a volume of the liquid equal to the volume of the submerged object. **Archimedes’ principle** indicates that the upward buoyant force that is exerted on a body immersed in a fluid, whether fully or partially submerged, is equal to the weight of the fluid that the body displaces. The net upward force on the object is the difference between the buoyant force and its weight. If this net force is positive, the object floats; if negative, the object sinks; and if zero, the object is neutrally buoyant—that is, it remains in place without either rising or sinking. The figure presented below demonstrates three different options in which the difference between the buoyant force $F_b$ and its weight $F_g$ is positive (I), zero (II), or negative (III). Keep in mind that $V_0=$ object volume, $\rho_f =$ the density of the fluid, $V_f=$the volume of the immersed part of the body in the fluid liquid, and $g=$ gravity.

![Diagram of buoyant forces](image)

6.10. David built a wooden raft. Evaluate the ratio of the greatest (maximum) weight it can hold on the water to the weight of the raft, if the raft is floating on the water. Take into account that the ratio (specific gravity) of the wood density to the water density is equal to $\frac{\rho_d}{\rho_w} = 0.8$.
6.11. David has constructed a raft from wood. The density of the wood in relation to the water density is equal to \( \frac{\rho_d}{\rho_w} = 0.5 \). Will this raft floating if the weight of the load is 0.5 of the weight of the raft?

6.12. Two cylinder containers (without liquid) with masses of \( M_1 \) and \( M_2 \), as shown in the figure below, are immersed in liquid. The immersed height of the first floating container with a radius of \( R_1 \) is equal to \( H_1 \), while that of the second floating container with a radius of \( R_2 \) is \( H_2 \). What is the ratio between \( H_1 \) and \( H_2 \)? It is assumed that the container masses are insignificant in comparison with the values \( M_1 \) and \( M_2 \).

**Hint:** According to Archimedes’ principle, any object that is wholly or partially immersed in a fluid is buoyed up by a force equal to the weight of the fluid displaced by the object. \( B = \rho_{\text{fluid}} \times g \times V_{\text{object}} \), where \( \rho_{\text{fluid}} \) = fluid density, \( V_{\text{object}} \) = volume of the immersed part of the object, and \( g \) = the acceleration of gravity. For a floating object, the buoyant upward force \( B \) is equal to the weight of the object, which is equal to \( W = m \times g \).

6.13. A hollow ball made from material with a density equal to \( \rho_M \) is floating in the liquid, which has density equal to \( \rho_L \). The external ball radius is \( R_E \) and the internal ball radius is equal to \( R_I \). What is the ratio between \( R_1 \) and \( R_E \) if \( \rho_{\text{material}}/\rho_{\text{liquid}}=2 \)?
6.14. The weight of the human body in air is 30 times greater than in seawater. What is the ratio of the density of seawater to the density of the human body?

6.15. It is known that the ratio of the density of the human body to the density of seawater is around 0.967 (the average human body density is 985 kg/m³, while seawater density is 1,020 kg/m³). Using this value, evaluate the ratio of the human body weight in air to the human body weight in seawater.

6.16. Hollow conical and spherical models made from metal float in liquid, as shown in the figure below. The radius of the cone’s base is equal to R₁ and the height of the cone is equal to the radius. Evaluate the ratio of the cone radius R₁ to the radius of sphere R₂ if it is known that both models are completely immersed, weighed in a container, and have the same weights.

```
Radius of the cone base is R₁  Sphere radius is R₂
```

```
<table>
<thead>
<tr>
<th>Cone</th>
<th>Sphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>F_b</td>
<td>F_b</td>
</tr>
<tr>
<td>mG</td>
<td>mG</td>
</tr>
</tbody>
</table>
Height of the cone is R₁
```

6.17. The first of four cylindrical containers, as shown in Figure 1 below, is filled with the water, while the other three containers are empty. When we connect all of the containers using pipes, as shown in Figure 2, the height of the water in all containers has a level of h. What is the height Q=h/H if the radius of the second cylinder is two times less than that of the first one, the radius of the third cylinder is two times less than that of the second one, and the radius of the fourth cylinder is two times less than that of the third one? When evaluating the level h, assume that the volume of water in the pipes is negligible in comparison with the cylinders’ volumes.
6.18. The value of the volume of liquid in each of three containers presents an integer number. The three numbers are consecutive terms of a geometric sequence. If you subtract 4 from the third term, then the three terms of geometric progression convert to the consecutive terms of arithmetic progression. If you subtract 1 from the second and third terms of the received arithmetic progression, then you again obtain consecutive terms of a geometric sequence. Find the integer numbers that determine the liquid volume in each container.

III. Electrical circuits, series, and parallel connections (Ohm’s law)

It is known that the total resistance of resistors in series is equal to the sum of their individual resistances:

\[ R_{\text{total}} = R_1 + R_2 + \ldots + R_n \]
Math for Physics (Problems)

The total capacitance of capacitors in series is equal to the reciprocal of the sum of the reciprocals of their individual capacitances:

\[
\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n}
\]

For resistors connected in parallel, \( \frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n} \), and for capacitors connected in parallel, \( C_{total} = C_1 + C_2 + \ldots + C_n \).

6.19. Evaluate the total resistance of 10 in identical resistors connected in series, each with a value of R.

6.20. Evaluate the total resistance of 10 resistors connected in series if their values are R, R/2, R/4... R/512.

6.21. Evaluate the total resistance of 10 identical resistors connected in parallel, each with a value of R.

6.22. Evaluate the total resistance of 10 resistors connected in parallel if their values are R, R/2, R/4... R/512.

6.23. Evaluate the total resistance \( R_{\Sigma(100)} \) of 100 resistors connected in parallel if their values are R, R/2, R/4, R/8...
6.24. Evaluate the total resistance \( R_{\Sigma(100)} \) of 100 resistors connected in parallel if their values are \( R, \frac{R}{3}, \frac{R}{5}, \frac{R}{7} \ldots \)

6.25. The total resistance of two identical, parallel-connected resistors is \( \frac{R}{2} \), where \( R \) is the resistance value of one resistor. Let us assume that the value of one resistor \( R \) is reduced by a factor of 2. What should be the value of the second resistor to obtain a total resistance equal to \( \frac{R}{3} \)?


6.27. Evaluate the resistance of the circuit shown in Figure 1 between points A and B.

\[ \text{Hint: Use the equivalence between circuits presented in Figure 2}. \]

\[ \]

Figure 1
6.28. The resistance of two parallel-connected identical resistors is \( R/2 \), where \( R \) is the resistance value of one resistor. Let a value of one resistor \( R \) be reduced by a factor of \( 2 \). What is the value of the third resistor that we need to add up in series with a parallel connection to obtain a total resistance equal to the original value of \( R/2 \)?

6.29. One of \( N \) identical parallel-connected resistors is increased by a factor of \( 2 \). Evaluate the total resistance of this circuit. Denote the resistance of one resistor by \( R \).

6.30. One of the \( N-1 \) identical resistors connected in parallel with each other is increased by a factor of \( 2 \). Then resistor with number \( N \) is connected in parallel to the circuit. What is the value of this resistance if the total resistance between points A and B is equal to \( R/N \)?
6.31. Suggest a series-parallel circuit connection for the identical resistors, each with value $R$, to obtain a total resistor value equal to $R_c = \frac{144}{11} R$.

6.32. Evaluate the capacitance value between points A and B for the circuit shown in the figure below if all capacitors are identical and have a capacitance value $C$.

6.33. Evaluate the capacitance value between points A and B for the circuit shown in the figure below.
6.34. Evaluate the capacitance value between points A and B for the circuit shown in the figure below if all capacitors are identical and have a capacitance value of C.

6.35. Suggest all possible connections of three identical capacitors with a value of C and evaluate their total capacitor value.

IV. The mathematics of lenses

6.36. An object is 50.0 cm to the left of a converging lens of +8.00 cm focal length. Determine the distance between the image and the lens.

Hint: Use lens equation \( \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \), which determines the relationship between the distance \( d_o = AO \) from the object to the converging lens, distance \( d_i = OC \) from the image to the converging
lens, and the focal distance \( f = F_0 \). Point \( F \) is the focal point, while \( 2F \) determines the location of double the focal distance. Parameters are demonstrated in the figure below.

6.37. A projector lens forms an image on a screen 10 times the size of its corresponding object. The screen is located 6 m from the lens. What is the required focal length of the lens?

**Hint:** The lens magnification is \( M = \frac{d_i}{d_o} \).

6.38. An object is 50 cm to the left of a converging lens of \( F = +8 \) cm focal length. The object has a height of \( h_o = 5 \) cm, as shown in the figure below. Determine the height of the image \( h_i \).

6.39. An object is 30 cm to the left of a converging lens of \( F_0 = 20 \) cm in focal length. The object has a height of \( h_o = 5 \) cm, as shown in the figure below. Determine the height of the image \( h_i \).
6.40. An object is 20 cm to the left of a converging lens of FO=45 cm focal length. The object has a height of \( h_o = 5 \) cm. as shown in the figure below. Determine the height of the image \( h_i \).

6.41. An object is placed 9 cm in front of a diverging lens. The lens has a focal length of 6 cm. Find the image distance \( OC \) and the magnification \( M = \frac{h_i}{h_o} = \frac{OC}{OA} \). 

\[ f=6 \text{ cm} \]
Hint: Use lens equation \( \frac{1}{d_o} + \frac{1}{d_i} = -\frac{1}{f} \), which determines the relationship between distance \( d_o \) (between the object and diverging lens), distance \( d_i \) (between the image and converging lens), and the focal distance \( f=FO \). Point F is the focal point, while \( 2F \) determines the location of double the focal distance. \( f=FO \). Parameters are demonstrated in the figure below. The magnification is \( M = \frac{h_i}{h_o} = \frac{OC}{OA} \). Parameters are demonstrated in the figure below.

6.42. The figure below shows an object \( O_1 \) (left part of the figure) that is placed in front of two thin lenses 1 and 2, with focal lengths of \( f_1=15 \text{ cm} \) and \( f_2=9 \text{ cm} \), respectively. The separation between lenses is \( L=8 \text{ cm} \). The distance \( p_1 \) is 6 cm from lens 1. What is the distance \( i_2 \) between the image and lens 2?

![Image of two lenses and an object](image1.png)

6.43. A simple magnifier makes an image appear at a near point distance from the eye viewer equal to 25 cm. What is the magnification power of the magnifier if it is constructed of a lens with a focal length of 5 cm?

Hint:
The near point distance $D$ is the nearest to the eye distance at which an object is clearly focused on the retina when the accommodation of the eye is at the maximum. The magnification power for the image at the near point is $M = 1 + \frac{25}{f}$, where $f$ is the magnifier’s focal distance. The attached figures show object and eye with and without magnifier.

6.44. The near point of a farsighted person is 100 cm (image placement). Girl places reading glasses close to her eye, and with them she can comfortably read a newspaper at a distance of 25 cm (object placement). What lens power is required?

**Hint:** Lens power given in diopters is expressed as $P = \frac{1}{f}$, where $f$ is in meters, and $1 \text{D} = 1 \text{ m}^{-1}$.

6.45. You are building a compound microscope with an objective lens of a focal length of 0.7 cm and an eyepiece lens of a focal length of 5 cm. You mount the lenses 18 cm apart. What is the maximum magnification of your microscope?

**Hint:** A schematic diagram of the compound microscope is given in the figure below.
It consists of one lens, the objective, which has a very short focal length $f_0 < 1$ cm and a second lens, the eyepiece, which has a focal length $f_e$ of a few centimeters. The two lenses are separated by a distance $L$ that is much greater than either $f_0$ or $f_e$. The object, which is placed just outside the focal point of the objective, forms a real, inverted image at $I_1$, and this image is located at or close to the focal point of the eyepiece. The eyepiece, which serves as a simple magnifier, produces at $I_2$ a virtual, inverted image of $I_1$. The overall magnification of the compound microscope is defined as

$$M \approx \frac{25L}{f_0 f_e}.$$

V. The law of reflection

*If a ray of light could be observed approaching and reflecting off of a flat mirror, then the behavior of the light as it reflects would follow a predictable law known as the law of reflection. The diagram below illustrates this law.*
In the diagram, the ray approaching the mirror is known as the **incident ray** (labeled \( I \) in the diagram). The ray leaving the mirror is known as the **reflected ray** (labeled \( R \) in the diagram). At the point of incidence where the ray strikes the mirror, a line can be drawn perpendicular to the surface of the mirror. This line is known as a **normal line** (labeled \( N \) in the diagram). The normal line divides the angle between the incident ray and the reflected ray into two equal angles. The angle between the incident ray and the normal is known as the **angle of incidence** (labeled \( \Theta_i \) in the diagram). The angle between the reflected ray and the normal is known as the **angle of reflection** (labeled \( \Theta_r \) in the diagram).

6.46. Two mirrors meet at angle \( \alpha \), as shown in the figure below. The beam of light strikes the horizontal mirror at angle \( \beta \). Find the angle \( \varepsilon \) between the incident and reflected light.

6.47. A plane mirror is illuminated by a light source located at \( 10^0 \) with respect to the normal line, as shown in the figure below. The angle between the mirror and horizon plane is equal to \( 30^0 \). Find the angle between the reflected ray and the horizon.
6.48. On a sunny day, it is necessary to illuminate the bottom of a draw-well. How should a flat mirror be replaced with respect to the horizon line to illuminate the bottom of the well if the incident rays fall on the mirror at an angle of $\alpha = 60^\circ$ to the Earth’s horizon? (See the figure below.)

6.49. The figure below shows a billiard table with dimensions of $20 \times 10$ (relative units). The cue ball is at a point with coordinates $(X=2, Y=20)$. The cue hits the ball at an angle of $45^\circ$ to the broad wall of the billiard table, as shown in the figure. How many times does the ball hit the walls before returning to the starting point?
Chapter 7

Probability and Statistics

Problems

7.1. There are two yellow apples and three red ones in the basket. You randomly select one apple. Evaluate the probability that you pick a red apple.

7.2. There are two yellow apples and three red ones in the basket. You randomly select one apple. Evaluate the probability that you take out a yellow apple.

7.3. There are five apples in the basket—two yellow and three red. You randomly select two apples in succession without replacing them. Evaluate the probability that both the first and second apples that you select from the basket are red.

7.4. There are two yellow apples and three red ones in the basket. You randomly select two apples in succession without replacing them. Evaluate the probability that the first and second apples that you select from the basket will both be yellow.

7.5. There are two yellow apples and three red ones in the basket. You randomly select two apples in succession without replacing
them. Evaluate the probability that the first apple selected from the box is red and the second is yellow.

7.6. There are two yellow apples and three red ones in the basket. You randomly select two apples in succession without replacing them. Evaluate the probability that the first apple selected from the box will be yellow and the second will be red.

7.7. There are two yellow apples and three red ones in the basket. You randomly select two apples in succession without replacing them. Evaluate the probability that the second and third apples selected from the basket will both be red if the first selected apple is yellow.

7.8. A basket contains five apples, of which one is spoiled and rest are good. If Victor randomly selects two apples from the basket in succession, what is the probability that the apples selected will include the spoiled apple?

7.9. The first basket contains one red apple and five yellow ones, while there are eight red and four yellow apples in the second basket. We take one apple from each of the first and second baskets. Find the probability that one of them is red and one is yellow.

7.10. A bag contains 10% small white cubes, 15% yellow ones, and rest are black. You select one cube at random. What is the probability that you will select a white or black cube?
7.11. There are three white balls and five black ones in the box. You take out two of them in succession at random without replacing them. Find the probability that they are different colors.

7.12. There are 5 white balls and 14 black ones in the bag. You randomly select two balls in succession without replacing them. What is the probability that first ball selected will be white and the second ball will be black?

7.13. There are three identical boxes with black and white balls. The first box contains one black ball and two white ones, the second contains one black and three white, and there are two black and two white balls in the third box. You select one of the three boxes at random and randomly select one ball from it. Find the probability $P$ that a white ball is selected.
7.14. There are six light bulbs in your apartment. The probability that each bulb will not break over the course of one year is 5/6. What is the probability $P$ that you will have to change at least half of the bulbs during one year if the bulbs operate independently from each other?

7.15. Tickets numbered 1 to 10 are mixed up and then a ticket is drawn at random. What is the probability that the number on the ticket drawn is a multiple of 3 or 5?

7.16. A box contains cards with the integer numbers from 30 to 40. You select one card from the box randomly. What is the probability that the number on the selected card will be divisible by 3?

7.17. There are three cards in the box. The name written on the first card starts with the letter A, that on the second card starts with the letter B, and that on the third card starts with the letter C. What is the probability that the cards will be selected from the box in alphabetical order?

7.18. There are four cards in the box. The name written on the first card starts with the letter A, that on the second card starts with the letter B, that on the third card starts with the letter C, and that on the fourth card starts with the letter D. What is the probability that the cards taken from the box will be in alphabetical order?
7.19. What is the probability $P$ that a randomly chosen integer numbered 1 to 100 is divisible by 2, if it is divisible by 3?

7.20. A student has to solve three multiple choice math problems. Each has one right answer and one wrong one. What is the probability that at least one of the answers will be correct if the probability of choosing a right answer is $1/2$?

7.21. A family has three kids. Evaluate the probability $P_{\Sigma}$ that at least one of them is a boy. The probability of boys or girls being born is 0.5.

7.22. A family has three kids. Evaluate the probability $P_{\Sigma}$ that at least one of them is a boy. The probability is 0.51 that a child being born is a boy.

7.23. A family has three kids. Evaluate the probability that all of them are boys. The probability of a boy being born is 0.51.

7.24. A family has three kids. Evaluate the probability that all of them are girls. The probability of a boy being born is 0.51.
7.25. A family has three kids. Evaluate the probability that two of them are boys. The probability of a boy being born is 0.51.

7.26. A family three kids. Evaluate the probability that the oldest and middle child are boys. The probability of a boy being born is 0.51.

7.27. Three brothers, Peter (P), Victor (V), and Alex (A), have to go to school. There are three different schools near the family residence: school #1, school #2, and school #3. Schools #1 and #2 have advanced math programs and school #3 has an advanced history program. Let us assume that each of the kids studies at a different school. What is the probability that Peter goes to a school with an advanced math program and Alex goes to the school with the advanced history program?

7.28. A biathlete has a probability of 4/5 of hitting the target. He shoots four times. What is the probability that he will hit the target at least three times?

7.29. Two hunters simultaneously shoot at a bear, which is killed by a single bullet. Find the probability $P_1$ that the bear was killed only by the first hunter, only the second ($P_2$), both at once ($P_3$), and neither the first nor the second ($P_4$), if the probability that the first and second hunter kills the bear is equal to 0.3 and 0.4, respectively.
7.30. Three hunters simultaneously shoot at a bear, and the bear is killed by a single bullet. Find the probability \( P \) that the bear was killed only by the third hunter if the probabilities that the first, second, and third hunter kills the bear are equal to 0.3, 0.4, and 0.5, respectively.

7.31. An electronic circuit includes three elements connected in series. The circuit is operating properly if none of the elements are not defective. The probabilities that the first, second, or third element is broken are \( P_1 = 0.1 \), \( P_2 = 0.2 \), and \( P_3 = 0.3 \), respectively. Find the probability \( P_\Sigma \) that the circuit is not operational.

7.32. The number of trucks traveling on the highway near a gas station is related to the number of cars passing along the same highway at a ratio of 3:2. The probability that a truck’s tank is filled with gas at the gas station is 0.1, while for a passenger car, this probability is 0.2. Find the probability that a truck or car will stop at the gas station to have the tank filled with gas.
7.33. The quality of manufactured parts can be controlled by a man or a woman. The probability that the part is inspected by a man is 0.6, while the probability that the inspector is a woman is 0.4. However, the probability that a man certifies the part as reaching the standard is 0.9, while the probability that a woman will do so is 0.8. Find the probability \( P \) that a randomly selected item is recognized as standard.

7.34. Twenty-five students were asked how many hours per day (on average) they watch TV. The results are presented in the table. However, one cell of the table is blank. It is known that the average TV-watching time is 1.8 hours. Find the appropriate number for the empty cell.

<table>
<thead>
<tr>
<th>Number of hours</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of students</td>
<td>1</td>
<td>9</td>
<td></td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

Determine the average time watching TV. Construct a pie chart of the distribution of the number of students depending on the time spent watching TV.

7.35. A student has written a number of tests. Based on the test results, he makes the following conclusion: If I receive 95 on the last test, my average score will be 90; if I receive 70 on the last test, my average score will be 85. How many tests were written by the student?

7.36. A student receives his half-year report card with 12 subject scores. His average score is 3.5 (5 is the maximum). He would like to raise his average score to 4 in the next semester. To do
this, he decides to increase the score by 1 point for a number of subjects. In how many subjects does he have to improve his results?

7.37. The table below indicates the noise level of an engine depending on the time. The noise level is shown in units called decibels. The data in the table are random values. Is it possible to evaluate the trend in the noise level over time using these data?

<table>
<thead>
<tr>
<th>Time</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noise</td>
<td>20</td>
<td>25</td>
<td>15</td>
<td>10</td>
<td>25</td>
<td>30</td>
<td>25</td>
<td>20</td>
<td>30</td>
<td>35</td>
<td>45</td>
<td>30</td>
</tr>
</tbody>
</table>

7.38. A store sells different products. The data in the table show the prices for different kinds of products in dollars. What is the mean absolute deviation of these data?

| X ($) | 34 | 42 | 12 | 23 | 56 | 39 | 68 | 11 | 59 | 48 |

7.39. What is the mean absolute deviation of the score achieved by a student who recorded the scores given below on 10 math quizzes?
Quiz scores: 68, 55, 70, 62, 71, 58, 81, 82, 63, 79.

7.40. Victor and two of his friends measured the height of their dogs (in cm). The heights were $X_1=24$, $X_2=8$, and one unknown measure, $X_3$. Evaluate $X_3$ if it is known that $0<X_3<16$ and the absolute median deviation of sequence $X_1$, $X_2$, and $X_3$ is 6.

7.41. A quadrilateral area is defined by four points with coordinates $X$ and $Y$. To estimate the size and shape of the site, several repeated measurements of these four coordinates were performed, and the results are presented below.
Assume the results in each table are random variables and use averaging procedure to evaluate the area of the lot.

<table>
<thead>
<tr>
<th></th>
<th>X1</th>
<th>0.5</th>
<th>3</th>
<th>2.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y1</td>
<td>0.6</td>
<td>3</td>
<td>2</td>
<td>2.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>X2</th>
<th>3.6</th>
<th>6.4</th>
<th>4.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y2</td>
<td>9</td>
<td>6</td>
<td>10</td>
<td>7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>X3</th>
<th>11.6</th>
<th>14.4</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y3</td>
<td>9</td>
<td>7</td>
<td>8.3</td>
<td>7.7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>X4</th>
<th>9.9</th>
<th>12.5</th>
<th>8.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y4</td>
<td>3</td>
<td>1.8</td>
<td>2.2</td>
<td>1</td>
</tr>
</tbody>
</table>

7.42. Property land in the triangle shape is defined by three points with their own coordinates X and Y on the plane. To estimate the size and shape of the site, Shawn carried out several repeated measurements of these three coordinates, and the results are presented below.

<table>
<thead>
<tr>
<th></th>
<th>X1</th>
<th>0.5</th>
<th>3</th>
<th>2.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y1</td>
<td>0.6</td>
<td>3</td>
<td>2</td>
<td>2.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>X2</th>
<th>3.6</th>
<th>6.4</th>
<th>4.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y2</td>
<td>9</td>
<td>6</td>
<td>10</td>
<td>7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>X3</th>
<th>10.1</th>
<th>9.9</th>
<th>12.5</th>
<th>8.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y3</td>
<td>3</td>
<td>1.8</td>
<td>2.2</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Assuming the results in each table are random variables and using the averaging procedure, evaluate the area of the lot.
7.43. Students in the physics classroom were asked to estimate the amount of the resistance $R$ by measuring current in the circuit shown in the figure below.

![Circuit Diagram]

A voltage $U=10\ \text{V}$ is applied to the circuit, and an ammeter measures current $I$ in the electrical circuit. Measurements are made several times, since the ammeter carries out measurements with a random error. The results of several measurements are summarized in the table below.

| Current $I$ | 1.2 | 0.8 | 1.4 | 0.9 | 1.1 | 0.6 |

Suggest an estimation of the resistance $R = \frac{U}{I}$ using the measured data.

7.44. Gas molecules in a closed cubic volume are in random motion. The average distance between molecules is equal to $L$. Assume that the volume of the cube is increased by two times while the number of molecules does not change. What is the average distance between the molecules now?

7.45. A point $M$ is selected at random on line segment $AB=L$. What is the probability that the ratio $AM/AB$ will be less than $1/3$?
7.46. A circle with a radius of $R$ is placed inside a square dart board with a length of $2R$, as shown in the figure below. The dart is equally likely to land on any point on the dart board. What is the probability $P$ that a dart thrown will land inside of the circle? Let $\pi \approx 3$.

7.47. Suppose that a skydiver has to drop onto a rectangular field that is 100 m by 125 m. He is equally likely to land anywhere in the field. However, the target region is a square with a length of 10 m and lies within the field. What is the value of the probability $P$ that the skydiver will land in the target region?

7.48. In the figure below, PQRS is a rectangle, and A, B, C, and D are the midpoints of the respective sides, as shown.

An arrow is shot at random into the rectangle PQRS. Calculate the probability that the arrow will strike a shaded region.

7.49. Alex and Shawn decide to meet at a bus stop between 11 and 11:05. Alex arrives at the bus stop at a random time between 10:55 and 11:00 and waits until Shawn comes. What
is the probability that Alex will wait for Shawn less than 3 minutes?
Chapter 8

Work Word Problems

Problems

8.1. One worker produces 20 car parts per hour, and the other produces 3 parts more. How many parts can both workers produce in 8 hours?

8.2. A cinema hall with N people inside has two doors. If only the first door is open, people will leave the hall in 20 minutes; if the second door is open and the first door is closed, the cinema hall will be empty in 30 minutes. How long will it take for N people to leave the building if both doors are open?

8.3. A pool can be filled by one pipe in 7 hours and by a second pipe in 8 hours. How long will it take to fill the pool using both pipes?

8.4. One inlet pipe can fill an empty pool in 5 hours, and a drain can empty the pool in 10 hours. How long will it take the pipe to fill the pool if the drain is left open?

8.5. Two workers together can complete the work in 6 hours. Working alone, the first worker can do the same work in 15 hours. How long will it take the second worker to do it alone?

8.6. Igor, Peter, and Vladimir are going to paint a fence. It is known that Igor and Peter can paint the fence in 9 hours, Peter and Vladimir in 12 hours, and Vladimir and Igor in 18 hours. How long would it take the three painters together to paint the fence?

8.7. The first company (A) can complete the production order 4 days faster than the second company (B). How long will it take each company to complete the work, if it is known that when working
together over 24 days, the companies can fill an order that is five times greater than the given one?

8.8. A field area of 2 hectares is divided into two equal-area lots. A farmer plows one portion of the field with a speed of $V_1 = 0.5$ hectares per hour, while his son plows the second portion of the field with a speed of $V_2 = 0.75$ hectares per hour. After the second plot is plowed, the son decides to help his father. How long will it take the father and son to plow the entire field?

8.9. An aquarium is filled with water coming in through two input tubes in 3 hours. How long will it take to fill an aquarium using the first input tube if this requires 2.5 hours less than to fill the aquarium through the second pipe?

8.10. Peter and Ivan complete the same test. Peter answers eight questions per hour, while Ivan answers nine. They both begin the test at the same time, and Peter finishes 20 minutes later than Ivan. How many questions are on the test?
Chapter 9

Geometry

Solutions and Answers

1.1. **Solution:** Assume that the screen width is 4x, and therefore the height is equal to 3x. Applying the Pythagorean Theorem gives $100^2 = (3x)^2 + (4x)^2$, or $x=20$. Thus, the width of the screen is 80 cm and the height is 60 cm.

**Answer:** The width is equal to 80 cm, and the height is 60 cm.

1.2. **Solution:** Based on the problem, the width of each step is equal to 800 mm. The area of step #2 (left to right) is equal to $390 \times 800 = 0.312 \text{ m}^2$, the area of step #1 is equal to $520 \times 800 = 0.416 \text{ m}^2$, and the front surface of step #3 is equal to $340 \times 800 = 0.272 \text{ m}^2$. Thus, the total area is 1 m$^2$.

**Answer:** 1 m$^2$.

1.3. **Answer:** The lengths of the quadrilateral are as follows: $540 \times 3/(3+4+5+6) = 90$ m; $540 \times 4/18 = 120$ m; $540 \times 5/18 = 150$ m; $540 \times 6/18 = 180$ m.

1.4. **Solution:** The sum of interior angles of a simple (and planar) quadrilateral is equal to $360^\circ$. Therefore, the angle values are as
follows: \(360 \times 3/(3+4+5+6) = 60^0\), \(360 \times 4/(3+4+5+6) = 80^0\), \(360 \times 5/(3+4+5+6) = 100^0\), \(360 \times 6/(3+4+5+6) = 120^0\).
Answer: 60\(^0\), 80\(^0\), 100\(^0\), 120\(^0\).

1.5. **Solution:** A hypotenuse \(BC\) of the right triangle \(ABC\) with legs equal to 120 m and 160 m is equal to 200 m (Pythagorean Theorem). Let us assume that the altitude \(AD=X\) and \(BD=Y\). Because triangles \(ABD\) and \(ABC\) are similar, \(160/120 = X/200\), \(120/120 = Y/200\) or \(X=AD=96\) m, \(Y=BD=72\) m, and \(DC=200-72=128\) m. Thus, the perimeter of triangle \(ABD\) is equal to 288 m, and the perimeter of triangle \(ADC\) is 384 m.

Answer: The perimeter of the triangle \(ABD\) is equal to 288 m and the perimeter of the triangle \(ADC\) is 384 m.

1.6. **Solution:** The ratio \(\frac{S_1}{S_2} = \frac{BD}{DC}\). According to the results of the previous problem, \(BD = 72\) m, \(DC = 128\) m. Thus, the ratio \(S_1/S_2 = 9/16\).

Answer: 9/16.
1.7. **Solution:** Values 20, 21, and 29 are a “Pythagorean Triple” set (a set of positive integers \(a, b,\) and \(c\) that fits the rule \(a^2+b^2=c^2\) [\(20^2+21^2=29^2\)]). Therefore, the triangular lot is a right triangle and its area is equal to \(20 \times 21/2 = 210\) m\(^2\).

**Answer:** 210 m\(^2\).

1.8. **Solution:** Let us divide the polygon into three right triangles and one rectangle, as shown in the figure below. The area of triangle ABF is 14 m\(^2\), the area of triangle GCD is 7 m\(^2\), the area of a triangle HDE is 14 m\(^2\), and the area of a rectangle FBCG is 56 m\(^2\). Thus, the total polygon square is 91 m\(^2\).

![Diagram of the polygon divided into triangles and a rectangle.]

**Answer:** 91 m\(^2\).

1.9. **Solution:** Let us assume that the length of one side of the lot is equal to \(X\). Then, the length of the other side is equal to \(X / 5\). According to the problem, \(X \times X / 5 = 2,000\), or \(X = 100\), \(X / 5 = 20\), and the perimeter is 240 m.

**Answer:** 240 m.

1.10. **Solution:** Let us say one side length is \(3X\) and the other side is equal to \(5X\). Then, according to the math problem, \(3X \times 5X = 135\) or \(X^2 = 9\) or \(X = 3\). Thus, \(3X = 9\) and \(5X = 15\).

**Answer:** 9 m and 15 m.

1.11. **Solution:** Let us say that one side of the equals \(X\), then the other side is \(X + 6\). Solve the equation \(X \times (X + 6) = 135\) or \(X^2 + 6X - 135 = 0\). \(X_1 = 9, X_2 = 15\).

**Answer:** 9 m and 15 m.
1.12. **Solution:** Let us say that X is one side, while the other is Y. Then, X+Y=180, X×Y=8,000. According to the math problem, X and Y are the root of the following quadratic equation $Z^2-180Z+8,000=0$. Solve $Z_1=X=100, Z_2=Y=80$.

**Answer:** One side is 100 m and the other side is 80 m.

1.13. **Solution:** Let us say that BE is a height lowered to the side AC in an equilateral triangle, and BF is a height lowered to the side of the CD in isosceles triangle CBD. Let us assume that the side of an equilateral triangle is equal to L. Then, AE=L/2 and the area of the triangle is equal to $\frac{L^2\sqrt{3}}{4}$. In an equilateral triangle CBD, angle $\angle BCD$ is 30°, so the height $BF=\frac{L}{2}$, the side is $CF=\frac{L}{2}\sqrt{3}$, and the area of the isosceles triangle is equal to $\frac{L^2\sqrt{3}}{4}$. Thus, the areas of these two triangles is equal, and their ratio is equal to 1.

**Answer:** 1.

1.14. **Solution:** Assume that one leg of the right triangle is equal to a, the other is b, and c= the triangle hypotenuse. Thus, we have a system of three equations with three unknowns.

\[
\begin{cases}
a^2 + b^2 = c^2 \\
a + b + c = 60 \\
\frac{ab}{c} = 12
\end{cases}
\]
Solution of the system: \(a=15\), \(b=20\), \(c=25\). Then the area of one triangle is 120 cm\(^2\), and the area of the second one is 90 cm\(^2\).

Answer: 120 cm\(^2\) and 90 cm\(^2\).

1.15. **Solution:** The triangular lot is shown in the attached figure. Points A, B, and C are defined by the coordinates A (0, -2), B (2, 0), and C (4, 2). The area of the triangle OBA is equal to 2, and the area of triangle OCB is equal to the area of the triangle OCD minus area of the triangle BCD or 4-2=2. The total area is 4.

Answer: 4.

1.16. **Solution:** The solution is shown below, where \(AC = CB\).

1.17. **Solution:** One of the solutions is presented in the attached figure.
According to the problem, OA=R. The length of the square can be calculated using the Pythagorean Theorem, as follows:

\[ OB^2 + \left( \frac{3}{2} OB \right)^2 = R^2, \text{ or } OB = \frac{2R}{\sqrt{13}} \approx 0.55R. \]

**Answer:** \( \approx 0.55R \).

1.18. **Solution:** Solution of this problem is identical to the solution of the previous problem.

**Answer:** \( \approx 68 \text{ cm}^2 \).

1.19. **Solution:** This can be solved using proof by contradiction, or proof by assuming the opposite (proof by contradiction is a form of indirect proof, that establishes the truth or validity of a proposition by showing that the proposition’s being false would imply a contradiction). Quadrilateral ABCD is shown in the attached figure. Assume that the sides AE=FD= \( \frac{1}{2} + \varepsilon \). Then, the area of a trapezoid is equal to
\[
\frac{3 + 2 \varepsilon}{2} \sqrt{1 - \left(\frac{1}{2} + \varepsilon\right)^2} \approx \frac{3}{2} \left(1 + \frac{2 \varepsilon}{3}\right) \sqrt{\frac{3}{4} \left(1 - \frac{2 \varepsilon}{3}\right)} = \frac{3}{2} \sqrt{\frac{3}{4} \left(1 - \frac{4 \varepsilon^2}{9}\right)}
\]

This expression is maximal if \( \varepsilon = 0 \), as required.

1.20. Solution: The number of boxes that fit on the truck is equal to \(2.4 \times 8 \times 3 \times 100 \times 100 \times 100 / 20 / 40 / 80 = 900\).
Answer: 900 boxes.

1.21. Solution: The width and height of the tiles are integer numbers. According to the Pythagorean Theorem, these values are 30 and 40 cm (or 40 and 30 cm), respectively, for a known diagonal equal to 50 cm. Therefore the number of tiles required to cover a floor with an area of 2 m\(^2\) is equal \(2 \times 100 \times 100 / 30 / 40 \approx 16.7\) (17).
Answer: 17.

1.22. Solution: According to the figure, \(AC/AS=2\) or \(SC/AS=3\). Triangles SAB and SCD are similar; therefore, \(CD/AB=SC/SA\) or \(CD=AB \times SC/SA=1.7 \times 3=5.1\) m.

Answer: 5.1 m.
1.23. **Answer:** 4 m or 3 m. (Pythagorean Theorem).

1.24. **Solution:** The attached figure explains how to find the distance AD. According to the problem, AB=11 m, BC=3.5 m, and CD=5.5 m. According to the Pythagorean Theorem,

\[ AD = \sqrt{AB^2 + BC^2 + DC^2} = \sqrt{11^2 + 3.5^2 + 5.5^2} \approx 12.8 \text{ m}. \]

**Answer:** \( \approx 12.8 \) m.

1.25. **Solution:** AB=4.2 m and BC=3.5 m; therefore, according to the Pythagorean Theorem, AC\( \approx 5.5 \). Because DC=5.5 m, angle \( \angle DAC \approx 45^\circ \).

**Answer:** \( 45^\circ \).

1.26. **Solution:** A closed smooth curve without self-intersections on a flat surface can be stretched, presenting a circle. The circumference
of the right and left skis differ by \( \Delta = 2 \times \pi \times 0.15 \approx 0.94 \) m and does not depend on the length of a smooth curve.

Answer: \( \Delta \approx 0.94 \) m.

1.27. Solution: The solution can be explained using the attached figure.

![Diagram](image)

According to the problem, \( OB = 4 \) m and \( OA = 1 \) m. Then, the vehicle height \( AB \) should be less than \( \sqrt{4^2 - 1} = \sqrt{15} \) (\( AB < \sqrt{15} \)). Obviously, the required number should be close to the number 4. Choose to verify \((3.8)^2 = 14.44\) and \((3.9)^2 = 15.21\). Thus, the answer is 3.8.

Answer: 3.8 m.

1.28. Solution: The forest area, together with water and uncultivated land, is equal to \( 208,000 \times \frac{53}{100} = 110,240 \) km\(^2\). Let us assume that the area of forest is equal to \( X \). Then, the area of the water is equal to \( X/6 \), and the area of uncultivated land is equal to \( X/3 \). That is, \( X + X/6 + X/3 = 110,240 \). Therefore, the forest area is \( X \approx 73.493 \) km\(^2\), and the water area is \( 12,249 \) km\(^2\).

Answer: \( \approx 73,493 \) km\(^2\).

1.29. Solution: Triangle ABC is a right triangle (\( \angle ABC = 90^0 \)), because the angle \( \angle BAD = 30^0 \) and \( \angle BCD = 60^0 \). Therefore, the length \( AB = 2BD = 20 \) m. According to the Pythagorean Theorem, length \( DC = BC/2 \) (\( \angle DBC = 30^0 \)) \( DC = \frac{BD}{\sqrt{3}} \) and \( BC = 2DC \approx 11.8 \) m; thus, \( AB + BC \approx 31.8 \) m.
1.30. Answer: $\approx 31.8$ m.

1.31. Solution: The area of sector $ABCD$ is equal to $\pi \times R^2/4$, since the angle $\angle AOC$ is $90^0$. The area of triangle $\triangle AOC$ is $R^2/2$, because triangle $\triangle AOC$ is a right triangle and $AO=OC=R$. Therefore, the cross-sectional area of the hangar (segment $ABC$) is $\pi \times R^2/4 - R^2/2 = R^2(\pi/2-1)/2 \approx 10.3$ m².

Answer: $\approx 10.3$ m².

1.32. Solution: According to the problem the cross-section of the garage hangar is part of a parabola $Y = A \times X^2 + B$. Obviously, for $X=0$, value of $Y$ must be equal to $h = 20$ m. Therefore, $B = 20$ m. For $X=5$ m value of $Y$ must be equal to 0. Therefore, $A=-4/5$. Value $A$ has a unit $1/m$. Thus, function $Y$ has the following form: $Y = -\frac{4}{5} \times X^2 + 20$. 

152
1.33. **Solution:** The cross-section of the garage hangar is part of a parabola $Y = -\frac{4}{5} \times X^2 + 20$ if $X=0$, $Y=h=20$ (height of the hangar); if $Y=0$, $X=L/2=5$ m (half of the hangar width). Therefore, the height is equal to 20 m and the width is 10 m.

Answer: Height=20 m; width=10 m.

1.34. **Solution:** Let us assume that ABC is arc of the circle centered at O. Represent arc ABC in the form of two line segments AB and BC. Find the circle radius $R = OC = OB = OA$, if we know that $AD = AC / 2 = 10$ m and the length of the line AB is approximately equal to the arc length of 10.01 m. From a right triangle ABD, we get the following:

$$BD = \sqrt{AB^2 - AD^2} = \sqrt{10.01^2 - 10^2} \approx 0.45.$$ 

Now, from the right triangle ODC, we can calculate $R^2 = (R - 0.45)^2 + 10^2$, or $R = \frac{100 + 0.45^2}{0.9} \approx 111.3$. 

Answer: $A=-\frac{4}{5}$; $B=20$. 

153
Answer: \( \approx 111.3 \text{ m.} \)

1.35. Solution: The right triangle BCD is isosceles, that is, BD = CD. On the other hand in a right triangle ACD length \( CD = \frac{1}{2} \times AC \). Thus, if \( CD = X, \ AC = 2X, \ AD = 1,000 + X \) or \( (2X)^2 = X^2 + (1000 + X)^2 \). From this quadratic equation, it can be found that \( X \approx 1,366 \text{ m.} \)

Answer: \( \approx 1,366 \text{ m.} \)

1.36. Solution: Comparing the triangles ACD and BCD and using the Pythagorean Theorem gives \( CD = 500 \times \sqrt{3} \approx 850 \text{ m.} \)
Answer: $\approx 850 \text{ m}$. 

1.37. **Solution:** Assume that the length $EC=X$ and $AD=Y$. It can be seen from the figure that $BC=2X$, $AC=2Y$, $4X^2 = X^2 + Y^2$ (1); $4Y^2 + (X+100)^2 + Y^2$ (2). Solving this system of two equations with two unknowns, we obtain $X = 50$ and $Y = 50 \times \sqrt{3} \approx 85$. Thus, the height of the second mountain is 150 m and the distance between the first and the second mountain is equal to 85 m.

Answer: The height of the second mountain is 150 m, and the distance between the first and second mountains is equal to 85 m.

1.38. **Solution:** The volume of liquid in the cylinder is $V_1 = \pi R^3$. The same volume of liquid in a cubic container is $V_2 = 4R^2 \times X$, where $X$ is the height of the liquid in the cube. Since $V_1 = V_2$, we have $R/X = 4/\pi$.

Answer: $R/X = 4/\pi$. 155
1.39. Solution: Water in the cone container occupies a volume equal to 
\[ V_1 = \frac{1}{3} \pi R^2 L. \] The same water in the cylindrical tank has a volume of 
\[ V_2 = \pi R^2 H, \] Because \( V_1 = V_2 \), 
\[ \frac{H}{L} = \frac{1}{3}. \]

Answer: \( \frac{H}{L} = \frac{1}{3} \)

1.40. Solution: One liter of water occupies a volume of 10 cm \( \times \) 10 cm \( \times \) 10 cm. Thus, the 64 l correspond to a cube with dimensions of 40 cm \( \times \) 40 cm \( \times \) 40 cm. The area of one face of the cube is equal to 1,600 cm\(^2\). The total area of the six faces of the cube is equal to 1,600 \( \times \) 6 = 9,600 cm\(^2\).

Answer: 9,600 cm\(^2\).

1.41. Solution: A cross-sectional area of the cornice is 16\( \times \)3 + \( \pi \times 5^2/4 + 7 \times 9 - \pi \times 4^2/4 \approx 118 \text{ cm}^2 \). The length of the cornice is 100 cm, so its volume is 118 \( \times \) 100 = 1.18 \( \times \) 10\(^4\) cm\(^3\).

Answer: 1.18 \( \times \) 10\(^4\) cm\(^3\).
1.42. **Solution:** The volume of the cylindrical part is \( \pi \times R^2 \times h \), while the volume of the half sphere is \( \frac{2}{3} \times \pi \times R^3 \approx 0.033 \text{ m}^3 \). The total volume is 0.25 m³. Therefore, \( h = \frac{(0.25 - 0.033)}{(\pi \times 0.25^2)} \approx 1.1 \text{ m} \).

**Answer:** 1.1 m.

1.43. **Solution:** The volume of the cylindrical part is \( \pi \times R^2 \times h \), while the volume of the hemisphere is \( \frac{2}{3} \times \pi \times R^3 \). According to the problem, they should be the same. Therefore, \( h = \frac{2}{3} R \).

**Answer:** \( h = \frac{2}{3} R \).
1.44. **Solution:** The crystal consists of two pyramids. The base plane of both pyramids is the same plane with an area equal to 12.25 cm. The height of one pyramid is 2.5 cm. A volume of one pyramid is \((1/3) \times 12.25 \times 2.5 \approx 10.2 \text{ cm}^3\). Therefore, the volume of the crystal from two identical pyramids is equal to 20.4 cm\(^3\).

Answer: 20.4 cm\(^3\).

1.45. **Solution:** The volume of the shape shown on the left side of the attached figure is equal to the cylinder volume shown in the right part of the figure. Because the diameter of the cylinder is 5 cm, the total volume is \(\pi \times 5^2 \times 15/4 \approx 294.4\).

Answer: \(\approx 294.4 \text{ cm}\).
1.46. **Solution:** The difference between the amounts of paint required to paint the outer and the inner surfaces of the tank is equal to \( \pi \times H \times (D_2 - D_1) \times Q \approx 90 \text{ g} \).

**Answer:** \( \approx 90 \text{ g} \).

1.47. **Solution:** To determine the base edge AB, we find the value of AO using the Pythagorean Theorem. Here, \( AO^2 = AS^2 - OS^2 \), and, therefore, \( AO \approx 162.9 \text{ m} \). Triangle AOB is a right and isosceles triangle. Therefore \( 2AO^2 = AB^2 \), or \( AB = AO \sqrt{2} \). Thus, \( AB \approx 230.4 \text{ m} \). The height SG of the triangle ASB is \( SG = \sqrt{OS^2 + OG^2} \); on the other hand, \( OG = \frac{AB}{2} \approx 115.2 \text{ m} \). Thus, \( SG \approx 186.5 \text{ m} \), and the total lateral surface area of the pyramid is \( 4 \times 230.4 \times 186.5 / 2 \approx 85,939 \text{ m}^2 \). The volume of the pyramid is \( (1/3) \times 230.4 \times 230.4 \times 146.7 \approx 2,595,815 \text{ m}^3 \).

**Answer:** The pyramid’s lateral area is \( \approx 85,939 \text{ m}^2 \), and its volume is \( \approx 2,595,815 \text{ m}^3 \).

1.48. **Solution:** Let us assume that the height OK of the pyramid with the base ABCD is equal to \( h \). The pyramid side AB=b, and the pyramid volume is equal to \( V_1 = \frac{S_1 \times h}{3} = \frac{b^2 \times h}{3} \), where \( S_1 = \text{area of the pyramid base} \). For a pyramid with a base AFGM, the volume is equal to \( V_2 = \frac{S_2 \times h}{3} = \frac{a^2 \times x}{3} \), where \( a = \text{side of a square AFGM} \) and altitude ON=x. According to the problem, ON=NK, or ON=OK/2, and planes EFGM and ABCD are similar. Because ON=OK/2,
S₁/S₂=4. Therefore, V₂/V₁=1/8 or the ratio of the volume of the shaded shape to V₁ is equal to 7/8 or 87.5%.

Answer: 87.5%.

1.49. Solution: Because the first and second cups have similar geometries, the ratio of their volumes is equal to

\[
\frac{V_1}{V_2} = \frac{D_1^2 \times h_1}{D_2^2 \times h_2} = \frac{0.5}{4} = \frac{1}{8} \quad (1)
\]

For similar cylinders, the relationship between h₁ and h₂ is equal to

\[
\left( \frac{h_1}{h_2} \right)^3 = \frac{V_1}{V_2} = \frac{1}{8}, \quad \text{or} \quad \left( \frac{h_1}{h_2} \right) = \frac{1}{2} \quad (2)
\]

Since h₁=10 cm, h₂=20 cm. Substituting equation (2) into (1) we obtain

\[
D_2^2 = 4D_1^2 \quad \text{or} \quad D_2 = 16 \text{ cm}
\]

Answer: D₂=16 cm, h₂=20 cm.

1.50. Solution: Let us assume that the volume of the vehicle is equal to V₁ and the volume of the similar model is V₂. The ratio V₂/V₁= (1/60)³, because the similar volumes are proportional to the linear dimensions in cube power. It is known that the body mass M = (body density) × (volume of the body), and because the vehicle and its model are made from the same material, the following holds:

\[
\frac{M_2}{M_1} = \left( \frac{1}{60} \right)^3, \quad \text{or} \quad M_2 = 1050/(60)^3 = 0.0049=4.9 \text{ g}
\]

Answer: 4.9 g.
1.51. **Solution:** According to the problem, the number of paper layers \( N \) wound around a cylinder is equal to the ratio of the roll thickness to the paper’s thickness. Thus, \( N = 100/0.5 = 200 \). The diameter of the first layer closest to the cylinder is equal to 20 cm, while diameter of the outer layer is equal to 30 cm. Therefore, the length of the first layer is \( 2\pi \times 20 \text{ cm} \), and the length of the last layer is \( 2\pi \times 30 \text{ cm} \). The length of the layer first to last changes as a linear function (arithmetic progression); therefore, the total paper length is equal to \( L = 200 \times 2\pi \times (20 + 30)/2 = 30,000 \text{ cm} = 300 \text{ m} \).

![Diagram](image)

Answer: 300 m.

1.52. **Solution:** The volume of the sphere is proportional to the cube of its diameter. If the diameter of the larger sphere is 4 times bigger than the diameter of the smaller one, the volume of the larger sphere is 64 times greater than that of the smaller one. That is, the volume of the Earth is 64 times greater than that of the Moon.

Answer: \( q = 64 \).

1.53. **Solution:** Suppose that the radius of the Earth is \( R_E \), and the radius of the planet Mercury is \( R_M \). The area of the Mercury is \( S_M = 4\pi R_M^2 \). Twice the surface area of Mercury is equal to \( S_{M_1} = 8\pi R_M^2 = 4\pi (\sqrt{2} R_M)^2 = 4\pi R_{M_1}^2 \), where \( R_{M_1} = \sqrt{2} R_M \). The ratio of the volume of Mercury to that of the Earth is proportional to the cube of the radius. That is,

\[
\frac{V_{M_1}}{V_E} = \frac{R_{M_1}^3}{R_E^3} = \frac{R_M^3 (\sqrt{2})^3}{R_E^3} = \frac{1}{6.4}.
\]

Thus, \( \frac{R_M}{R_E} = \frac{1}{\sqrt{2}} \sqrt[3]{\frac{1}{6.4}} \approx 0.38 \).

Answer: \( \approx 0.38 \).
1.54. **Solution:** Based on the figure, it can be seen that the maximum distance between people located in the hot air balloon and a point on the Earth’s surface is equal to the length $MT$. Angle $\angle OTM = 90^0$. $MO = 6,370 + 4 = 6,374$ km, and $OT$—the Earth’s radius—is 6,370 km. Applying the Pythagorean Theorem gives the following:

$$MT^2 + T^2 = MO^2; \quad MT^2 = MO^2 - OT^2; \quad \text{or} \quad MT \approx 225.8 \text{ km}.$$

**Answer:** $MT \approx 225.8 \text{ km}$.

1.55. **Solution:** A full $360^0$ turn corresponds to 24 hours, so the Earth rotates $60^0$ in 4 hours.

**Answer:** $60^0$.

1.56. **Solution:** The earth completes one revolution of $360^0$ around the sun every year or 365 days or $365 \times 24 = 8,760$ hours. Thus, 4 hours corresponds to $\approx 0.16^0$.

**Answer:** $\approx 0.16^0$.

1.57. **Solution:** The length of the equator $L = 2 \times \pi \times R$, where $R$ is the radius of the Earth. Hence, the radius $R = L / (2\pi)$. According to the statement of the problem, the length of the equator is equal to $L = 40,000,000$ m or 40,000 km. Thus, $R = 40,000 / (2\pi) \approx 6,369$ km.

**Answer:** $\approx 6,369$ km.
1.58. Solution: Since the circumference along the equator is L≈40,000 km and the length of the arc ACB is 6,679 km, angle \( \angle AOB \) is \( 360 \times 6,679/40,000 \approx 60^\circ \). Thus, triangle \( \triangle AOB \) is an equilateral triangle, that is, the length AB is equal to Earth radius R. The radius of the Earth can be calculated from the known value of the circumference \( L=2 \times \pi \times R \). \( AB=R=L/(2 \times \pi) \approx 6,369 \) km.

![Diagram of Earth and Triangle AOB](image1)

Answer: \( AB \approx 6,369 \) km.

1.59. Solution: Assume that the distance from the Earth to the Moon is equal to R. Refer to the attached drawing: The Earth is located at the center of the circle at point O, while the Moon is represented by the chord-arc AB=3,400 km. The full circumference \( 2\pi R \) corresponds to the angle \( 360^\circ \), and the 3,400 km arc corresponds to an angle of \( 30'=1/2 \) degrees. Therefore, the following expression can be given: \( R=3,400 \times 2 \times 360/ (2 \times \pi) \approx 408,000 \) km.

![Diagram of Earth and Moon](image2)

Answer: \( \approx 408,000 \) km.
1.60. **Solution:** The linear length scale on the globe is 1:40,000,000 so the area of Russia on the globe is 17,000,000/40,000,000/40,000,000×(10^{10})≈106 cm^2.

**Answer:** ≈106 cm^2.

1.61. **Solution:** The linear length scale on the globe is 1:40,000, so the area of Canada on the globe is 10,000,000/40,000,000/40,000,000×(10^{10})≈62 cm^2.

**Answer:** 62 cm^2.

1.62. **Solution:** The attached figure shows the radius AB of the desired circle R. Based on the geometry of the figure, we have OC=OB and

$$AB = \frac{OC}{\sqrt{2}} \approx 0.707 \times OC.$$ 

Therefore, the length of the circle at a latitude of 45° is approximately equal to 0.707×40,000 = 28,280 km.

**Answer:** 28,280 km.

1.63. **Solution:** Assume that CE is the circumference of the Earth, LR is the length of the rope, and RE and RR are the radius of the Earth and the rope shaped in a circle, respectively. We have to estimate H=RR - RE. Denote as X the value we need to add to the length of the rope (in our case X=1 m). The relationship between the circumference C of a circle and its radius is given as R = C/2π. Thus, the following can be expressed:

$$C_E = 2\pi R_E \quad C_E + X = 2\pi \left( R_E + H \right),$$

where X=1 m, or $$X = 2\pi H$$

and $$H = \frac{1}{2\pi} \approx 16$$ cm. This means that a mouse can crawl under the rope.
Answer: Yes.

1.64. Solution: As shown in the attached figure, the moon travels in a circular orbit with a center that coincides with the Earth’s center, and a radius that equals the distance from the Earth’s center to the center of the Moon. The radius of this circular orbit, then, is 6,378+382,500+1,737=390,615 km. The circumference of this circular orbit is the distance traveled by the Moon as it orbits Earth. Thus, the moon travels $2\times\pi\times r = 2\times\pi\times 390,615 = 781,230\pi$ km. Because the Moon’s speed is equal to 3,680 km/h, the distance traveled by the moon in 24 hours is $3,680\times 24 = 88,320$ km. Therefore, the Moon completes its orbit around the Earth in approximately $781,230\pi \div 88,320 \approx 27.3$ days.

Answer: $\approx 27.3$ days.
Chapter 10

Percentage and Finance

Solutions and answers

2.1. **Solution:** A year ago, the price of a home was $102,000; today it is $102,000 \times 1.02 = $104,040.
    
    **Answer:** $104,040.

2.2. **Answer:** 1.1.

2.3. **Answer:** $125.

2.4. **Answer:** $120.

2.5. **Answer:** 1,352 m.

2.6. **Answer:** 6.112 ha.

2.7. **Solution:** After reduction, the salary became $2000 \times (1-0.2) = 1,600$ dollars. After the increase, the salary was equal $1600 \times (1+0.2) = 1,920$ dollars.
    
    **Answer:** $1,920.

2.8. **Answer:** 50%.

2.9. **Solution:** In the current year, the number of sales should be $2000 \times (1+0.25) = 2,500$; the price of the unit will be equal to $160 \times (1+0.1) = 176$ dollars. The number of sales with returned merchandise will represent $2500 \times (1-0.05) = 2,375$. The total value of sales is $2375 \times 176 = 418,000$ dollars.
    
    **Answer:** $418,000.

2.10. **Solution:** Following from the problem, the number of students is $10+14+4+12=40$. Victor got $4 \times 100/40 = 10\%$ of votes, Shawn $12 \times 100/40 = 30\%$, and Victor + Shawn = $(14+12) \times 100/40 = 65\%$. 
    
    167
Answer: 65%.

2.11. **Solution**: Let’s assume that $A$ represents the salary increase, while $X$ is the initial salary. Keep in mind that a 10% increase corresponds to increasing by factor $(1+0.1)$. Thus, $X\times(1+0.1)\times(1+A)=X\times(1+0.32)$. Therefore, $A=0.2$ and does not depend on the value of $X$. Thus, during the second part of the year, the salary has increased by 20%.

Answer: 20%.

2.12. **Solution**: If in two years, the final deposit is equal to $10,816$, then the initial investment deposit is $10816/1.04/1.04=10,000$.

Answer: $10,000$.

2.13. **Solution**: In the first case, a watermelon flesh (excluding skin) costs $10/V$ dollars. For the second type of watermelon, the flesh costs $7/(V-0.2V)=7/(0.8V)<10/V$. Therefore, it is preferable to buy the watermelon with the thin skin.


2.15. **Solution**: The price $P$ is $A\times(1-0.2)=0.8A$. The price $A$ is $0.8Q\times(1+0.6)=1.28Q$. Therefore, $A/P=1.28/0.8=8/5$.

Answer: $A/P=8/5$.


2.17. Answer: 40%.

2.18. **Solution**: $12\to7.5\%$; $x\to100\%$. $X=12\times100/7.5=160$.

Answer: 160.

2.19. **Solution**: The final sales increase in March in comparison with January is equal to $1.2\times1.2=1.44$.

Answer: 44%.

2.20. **Solution**: If the original price is $A$, then the final sales increase can be calculated as follows: $B = A \times (1 + 0.05)^{10} \approx 1.63$. 
Answer: 63%.

2.21. Answer: 60%.

2.22. Solution: Suppose we have X grams of milk with 2% fat and Y grams of milk with 18% fat. According to the problem, X+Y=1,000; 0.02×X+0.18×Y=50. The solution of the two equations with two unknowns is: X=812.5, Y=187.5.
   Answer: 812.5 g of 2% fat milk and 187.5 g of 18% fat milk.

2.23. Solution: Let’s assume that the mass of the first alloy is X. Then, the mass of the second alloy is equal to X+3. The mass of the copper in the first alloy is 0.1X, and in the second composition, it is 0.4(X+3). The third composition, which is the sum of the first and second ones, contains 30% copper. Thus, 0.1X+0.4(X+3) = 0.3(X+X+3). The solution for this equation gives X=3 kg. Therefore, 2X+3=9.
   Answer: 9 kg.

2.24. Solution: In accordance with the problem, the initial alloy has 4 kg of copper and 6 kg of tin. To obtain equal amounts of copper and tin, it is necessary to add 2 kg of copper to the alloy.
   Answer: 2 kg.

2.25. Solution: Four liters of solution contains 1/5 l of alcohol and 19/5 l of water. The new solution contains 1+19/5=24/5 l of water and 1/5 l of alcohol. Thus, the concentration of the new solution is (1/5)/(24/5+1/5)×100=4%.
   Answer: 4%.

2.26. Solution: Let X be the volume of 30% acid solution in 300 ml and Y be the volume of 50% acid solution in 300 ml. Then, X+Y=300 ml (1). We now write an equation that expresses that the total acid in the final 300 ml is equal to the sum of the amounts of acid in X and Y, 0.4×300=0.3×X+0.5×Y (2). Solving the system of two equations with two unknown, we obtain X=150 and Y=150.
   Answer: First solution amount is 150 ml and second solution amount is 150 ml.
2.27. **Solution:** Let us assume that the original price of the car is \( A \). According to the problem, 0.65\( A \) was paid when a company bought the car and the income came out to \( A \times 0.65 \times 1.5 = 0.975A \) when the car was sold. Therefore, the company did not profit in comparison with the original car price.

**Answer:** No.

2.28. **Solution:** At the end of the first year, he has the principal plus the interest on the principal:

\[
P_1 = 1000 + 0.03 \times 1000 = 1000(1+0.03).
\]

At the end of the second year, he has the principal \( P_1 \) plus the interest on \( P_1 \):

\[
P_2 = P_1 + 0.03 \times P_1 = P_1(1+0.03).
\]

Substitute \( P_1 \) by \( 2000 \times (1+0.03) \) above to obtain:

\[
P_2 = 1000 \times (1+0.03)^2.
\]

Continuing with this process, it can easily be shown that at the end of the 10th year, the principal is given by:

\[
P_{10} = 1000 \times (1+0.03)^{10} \approx $1,340.
\]

**Answer:** $1,340.

2.29. **Solution:** Based on the previous problem’s solution, the interest earned by Victor is equal to \( q_1 = X \times (1.05)^{10} - X \), where \( X \) = initial contribution. Meanwhile, Shawn’s earned interest is \( q_2 = Y \times (1.07)^8 - Y \). Thus, the ratio \( q_1/q_2 = ((X/Y) \times ((1.05)^{10} - 1))/((1.07)^8 - 1)) \approx 0.88 \times (X/Y) \). According to the problem, \( q_1 = q_2 \), or \( X/Y \approx 1/0.88 = 1.14 \).

**Answer:** 1.14.

2.30. **Solution:** The interest to pay is given by the following:

\[
\text{Interest} = 800 \times 0.1 + 880 \times 0.1 = 168.\]

The total to repay is \( 800 + 168 = 968 \).

**Answer:** $968.

2.31. **Solution:** Let’s assume that John payment at the end of the first year is equal to \( X \) dollars. Then, the balance at the end of the second year is \( (1090-X) \times 1.09 \) dollars. According to the math problem \( (1090-X) \times 1.09 = X \). Thus, \( X \approx 568.5 \) dollars.

**Answer:** \( \approx $568.5 \).
2.32. **Solution:** A 12% down payment corresponds to $504. Thus, Mary needs to repay a balance of 4200-504=3,696 dollars during the two years. However, because of bank service charges, she pays 195×24=4,680 dollars. Therefore, the finance charge of her loan is equal to 4680-3696=984 dollars.

Answer: $984.

2.33. **Solution:** A 10 kg watermelon with 99% water has 9.9 kg water. After evaporation, the watermelon contains 98% water. Let us assume that the weight of water in the evaporated watermelon is X. Then, X/(X+0.1)=0.98. So, X=4.9, or X+0.1=5.

Answer: 5 kg.

2.34. **Solution:** Fresh mushrooms contain 10% of dry substance. Thus, 11 kg of fresh mushrooms have 1.1 kg of dry substance. To get dry mushrooms, we need to add X water, so X/(X+1.1)=0.12. X=0.15 kg or 1.1+0.15=1.25 kg.

Answer: 1.25 kg.

2.35. **Solution:** Let us assume that X=the basket weight and Y=the suitcase weight. Then, (5X+2Y)×2=3X+10Y, and X=0.857Y.

Answer: ≈86%.

2.36. **Solution:** The customer has to return $400,000 + 3% interest paid over 25 years. According to the problem formula for monthly payment

\[ M = \frac{P_i}{1 - (1+i)^{-n}} \approx$1897 \]  

(i=0.03/12=0.0025, n=12×25=300, P=$400,000). At the end of the term, the payment becomes 1897×12×25=569,100 dollars. as follows: 12,000×25/2=150,000 dollars. Thus, taking the bank’s $400,000, the buyer will repay $569,100 after 25 years.

Answer: $569,100.

2.37. **Solution:** Let us assume that the boys bought X kg of apples, 1.5X of pears, and according to the problem, (X+0.6X) kg of oranges. The total amount of fruit is equal to X+1.5X+(X+0.6X)=12.3. Thus, X=3, 1.5X=4.5, and X+0.6X=4.8.

Answer: Apples, 3 kg; pears, 4.5 kg; oranges, 4.8 kg.
2.38. Solution: The surface area of the sphere is $4\times \pi \times R^2 \approx 1,256 \text{ cm}^2 = 0.1256 \text{ m}^2$. Next, you need to add another 8% to obtain 0.1256$\times$1.08$\approx$0.1356 $\text{m}^2$. 
Answer: $\approx$0.1356 $\text{m}^2$.

2.39. Solution: Let us assume that we have initially $1$, which corresponds, for example, to $A$ rubles. After increasing the exchange rate by 10%, $1$ is worth $A\times1.1$ rubles. Therefore, 1 ruble is worth $1/(A\times1.1)\approx 0.909/A$ dollars. Thus, the cost of the ruble decreased by $(1-0.909)\times100=9.1\%$.
Answer: $\approx$9.1\%.

2.40. Solution: Let us assume that there are $X$ boys in the school. Thus, the amount of girls is 0.6$X$. The total number of students is $X+0.6X=1.6X$. Therefore, the percent of girls is equal to $(0.6/1.6)\times100=37.5\%$.
Answer: 37.5\%.

2.41. Solution: Let us assume that the initial salary of the wife is $X$, and $Y$ is the initial salary of her husband. Then, according to the statement of the problem, $2X+Y=(X+Y)\times1.45$; or $X=45\times Y/55$. On the other hand, $(X+2Y)=(X+Y)\times Z$, where $Z$ is the relative increase in the total income of the family if the husband’s salary is doubled. Solving the system of two equations defines $Z=1.55$.
Answer: 55\%.

2.42. Solution: Let us assume that initially, Greg has $A$ milliliters of water. Thus, Peter has 1.1$A$ milliliters of water. After drinking, Greg has 0.98$A$, and Peter has 0.89$\times$1.1$A=0.979A$ milliliters. Thus, Greg has more water than Peter.
Answer: Greg.

2.43. Solution: The area of pizza per dollar in the first size is equal to $\approx 50\pi$; for second size it is $62.5\pi$; about $57.5 \pi$ for the third size; and $45\pi$ for the fourth size. Therefore, it is preferable to buy the second pizza size.
Answer: It is preferable to buy the second pizza size.
2.44. Answer: \( \frac{k-1}{k} \times 100 \).

2.45. Solution: Because the payment is a linear function of the temperature, 30 days of heating at a temperature of \( -15^\circ \) costs $160. Thirty days include \( 30 \times 24 \) hours. Eight hours of heating with a linear dependence of the value from time costs $160/(30/3) \approx 1.8.
Answer: $1.8.

2.46. Solution: The difference between the two car prices is $10,000. During the first year, the usage costs for the gas car are equal to \( A_1 = 1,500 \); in the second year, \( A_2 = 1,500 \times 1.05 \); in the third year, \( A_3 = 1,500 \times 1.05 \times 1.05 \); and so on. The numbers \( A_1, A_2, \) and \( A_3 \) are terms of geometric progression. The sum of the 10 members (in 10 years of operation) is \( 1500 \times (1.059-1)/(1.05-1) \approx 1500 \times 11.03 \approx 16,540 \) dollars. The total cost of electricity to charge the electric car for 10 years is equal to \( 500 \times (1.19-1)/(1.1-1) = 6,790 \) dollars. Thus, the gas car costs $30,000+16,540=46,540 dollars, while the electric car’s price is $40,000+6790=46,790 dollars. Thus, the car costs are almost the same if both of them run for 10 years.

2.47. Solution: Expenses for the car with a gas engine (together with the price of the car) are as follows: $\$(20000+12 \times 5 \times (1800 \times 0.12+600 \times 0.08)) = 35,840$. The car that runs on diesel costs $\$(25000+12 \times 5 \times (1800 \times 0.112+600 \times 0.07)) = 39,616$. The expenses for the gas car are less than the expenses for the diesel car.
Answer: The gas car is preferable.

2.48. Solution: Let us estimate the fuel expenses for the gas and diesel cars after 5 years of usage. Over this time, gas costs $\$12 \times 5 \times (1800 \times 0.12+600 \times 0.08) = 15,840$, while diesel costs $\$12 \times 5 \times (1800 \times 0.112+600 \times 0.084) = 15,120$. As we can see, fuel expenses are almost the same. Because the price of the gas car is cheaper than that of the diesel car, it is preferable to buy a gas car for 5 years of usage.

2.49. Solution: Let us assume that the out of town monthly car mileage is \( X \). Then, the total gas car expenses over 5 years (including the car price) are \( A=20000+12 \times 5 \times (2000 \times 0.12+X \times 0.08) \). The diesel car
expenses are \( B=25000+12\times5\times(2000\times0.14\times0.8+X\times0.14\times0.5) \). If \( A=B \), then \( X \approx 673 \) km. So, if the out of town monthly car mileage \( X \leq 673 \) km, the gas car is preferable; meanwhile, if \( X > 673 \) km, then the diesel car is cheaper for usage.

2.50. Solution: Expenses for the first car are \( $(12\times3000+20000) = $56,000 \), while for the second one, they are \( $(14\times0.7\times3000+22000) = $54,400 \). Therefore, the second car, which runs on diesel, is more cost effective.  
Answer: The second car.

2.51. Solution: It follows from the problem that the desired number of days cannot be more than six; otherwise, the remaining amount will not be an integer, since \( 100000/2^7 \) is not an integer, while \( 100000/2^6 \) is an even integer. Now, a simple iteration of values \( 55000-n\times15000 \) shows that the sum of the first and second product sellers will be equal for \( n=2, 100000/2^2 = 25000 \) and \( 55000-n\times10000 = 25000 \).  
Answer: 2.

2.52. Solution: Let us assume that Daniel has \( $X \) in his bank account and his sister has \( $Y \). According to the problem, \( X+Y=28000 \) and \( X-Y=4000 \). Therefore, \( X=16,000 \). So, Daniel has access to \( 16000+3000=19,000 \) dollars. This is not enough to buy a car, since Daniel’s money plus \$3,000 from his sister is less than \$20,000.  
Answer: No.

2.53. Solution: The cost of transportation with any company is equal to the price of 100 km transportation \( \times (1000/10) \times (\text{number of trips=40/capacity of the car}) \).  
Shipping with company #1 will cost \( $(10\times10\times65) = $6,500 \); with company #2 it will cost \( $(10\times40/2.5\times40) = $6,400 \); with company #3 it will cost \( $(10\times50/2\times40) = $10,000 \); and with company #4 it will cost \( $(10\times25/1.5\times40) \approx $6,667 \).  
Answer: Company #2.

2.54. Solution: The average price is equal to the sum of all possible prices divided by the number of different types of sausage, that is, 15. Thus, the cost of all sausage varieties is equal to \( 10.15\times15=152.25 \). This value is not an integer, while each sausage
value is an integer number according to the problem. This means that the average price cannot be a number equal to $10.15.  
\textbf{Answer: No.}

2.55.  \textbf{Solution:} The sum of all goods except for the notebook is $5.27. This is an odd number. However, according to the problem, the total cost of four identical rolls of tape, eight identical notebooks, and two pens must be expressed as an even number. Hence, the seller made a mistake.

2.56.  \textbf{Solution:} Since the number 140 ends by digit 0, sum of the product of 16 to some integer and the product of 17 to the integer has to be divisible by 10. Such a combination may be as following: 16\times2+17\times4+40=140.  
\textbf{Answer:} Yes.

2.57.  \textbf{Solution:} The total weight value of 450 g packs has to end in 00; otherwise, the total weight of the red and blue 100 g packs will not be equal to 2,500 g. Therefore, only four red packs can be included to provide an appropriate solution (450\times4=1,800). Thus, the decision is as follows: four red packs of 450 g and 7 blue packs of 100 g.  
\textbf{Answer:} Four red packs of 450 g and 7 blue packs of 100 g.

2.58.  \textbf{Solution:} Let us assume that the area of the whole field is equal to A. The 16\% of the field that is unploughed is equal to 8 ha. Thus, the whole area of the field is A=100\times8/16=50 ha.  
\textbf{Answer:} 50 ha.

2.59.  \textbf{Solution:} Twenty-five tons of laminate with a weight 5 kg for one pack corresponds to 5,000 packages. Therefore, when buying from supplier A, the construction company will pay $25,500 for the laminate, while $26,000 to supplier B and $27,000 to supplier C. Together with delivery costs, we obtain $26,500 for company A, $26,400 for company B, and $27,100 for company C. The cheapest supplier is B.  
\textbf{Answer:} B.
2.60. **Solution:** According to plan #1, the customer will pay $150; according to plan #2, the customer will pay $(20+100)=$120, and according to plan #3, the customer will pay $70. Thus, plan #2 is the most economical.

**Answer:** #2.

2.61. **Solution:** Let us assume that the required number of minutes is equal to N. Then, $N \times 0.15 = 20 + (N - 600) \times 0.25$. N=1,300. E. If the number of minutes spoken in the month is less than 1,300, plan #1 is most economical; with more than 1,300 minutes but less than 1,520 minutes, the most beneficial plan is #2; and for more than 1,520 minutes, the most beneficial plan is #3.

2.62. **Solution:** If customer uses plan #1 he pays $10 \times 10 + 10 \times 50 + 2 \times 10 + 2 \times 120 = 860$ dollars per year. Plan #2 corresponds to $30 \times 10 + 10 \times 10 + 2 \times 30 + 2 \times 70 = 600$ dollars. If customer uses plan #3 he pays $840. The other plans correspond higher dollar per year. Thus, plan #2 is the cost effective.

**Answer:** Plan #2 is preferable.

2.63. **Solution:** According to the problem, plan #2 or plan #3 is preferable for the customer. If we select plan #2, the payment will be as follows: For the first 10 months, the customer pays $30 \times 9 + 10 \times 9 = 360$ dollars. For the last 3 months, the user will pay $30 \times 9 + 80 \times 3 = 510$. The total amount is equal to $870$. If the user orders plan #3, the total payment for the year is equal to $70 \times 12 = 840$ dollars. Thus, plan #3 is preferable.

**Answer:** Plan #3 is most cost effective.

2.64. **Solution:** Assuming that the jam jars have cylindrical shapes, estimate their volumes. The volume of the taller jar is $V_1 = \pi \times R^2 \times H$, the volume of the shorter jar is $V_2 = \pi \times (2R)^2 \times \frac{H}{2} = 2\pi \times R^2 \times H$. Thus, the second jar holds twice as much jam as first one, while only costing 1.5 times more. Therefore, it is preferable to buy the second jar.

**Answer:** The second jam jar is more cost effective.
2.65. **Solution:** Let us assume that \( X \) is the area growing corn, and \( Y \) is the area growing sugar beets. Then, the conditions of the problem can be expressed by the system \( X + Y = 100 \), \( 50X + 20Y \geq 3200 \). We express \( Y \) from the first equation as follows: \( Y = 100 - X \), and substitute \( Y \) into the second equation: \( 50X + 20(100 - X) \geq 3200 \), \( (30X \geq 1200) \) or \( (X \geq 40) \).

**Answer:** The smallest area that can be planted with corn, is 40 ha.

2.66. **Solution:** According to the problem, 5 l of the mixture contains 4.5 l of water and 0.5 l of paint.

**Step 1:** After the first transfusion, the second bottle contains 5+4.5=9.5 l of water and 0.5 l of the paint. The first bottle contains 4.5 l of water and 0.5 l of paint. **Step 2:** Five liters of the liquid in the second bottle now contains 4.75 l of water and 0.25 l of paint. This mixture volume is transfused into the first bottle, and now the second container contains only 4.75 liters of water and 0.25 l of paint. The first bottle has 4.5+4.75=9.25 liters of water and 0.5+0.25 liters of paint.

**Step 3:** Take 5 l of the mixture (4.675 liters of water and 0.325 liter of paint) from the first bottle and transfuse it into the second container. Then, we obtain 9.425 l of water and 0.575 l of paint in the second bottle. The first bottle contains 4.675 l of water and 0.325 liters of paint. Thus, the first bottle includes 0.325×100/5=6.5% paint, and there is 0.575×100/10=5.75% paint in the second bottle.

**Answer:** The first bottle contains 6.5% paint and the second bottle contains 5.75% paint.

2.67. **Solution:** Initially, the first bottle has a liquid volume of 10 liters equal to 90% water and 10% paint. Five liters of the mixture (4.5 l of water and 0.5 l of paint) is transfused into the second bottle. As a result, there is now 9.5 l of water and 0.5 l of paint in the second bottle. Half of this amount, that is, 4.75 l of water and 0.25 l of paint, is transfused into the third bottle. Therefore, there is 7.25 l of water and 0.25 l of paint in the third bottle. Finally, 3.625 l of water and 0.125 l of paint is poured into the fourth bottle, and the fourth container has 4.875 l of water and 0.125 l of paint. Thus, the fourth bottle contains 5 l of liquid, made up of 97.5% water and 2.5% paint.

**Answer:** 2.5%
2.68. **Solution:** The required number of percent $X$ can be found from the following equation: $(1 + X)(1 + X) = 1.21$. Solving the quadratic equation gives $X = 0.1$.

**Answer:** 10%.

2.69. **Solution:** According to the statement of the problem, $(1 + 0.25)^n > 3$, where $n$ = the required number of years. That is, it is necessary to solve the inequality $1.25^n > 3$. By simple brute force, find $n = 5$, because $1.25^4 < 3$ and $1.25^5 > 3$.

2.70. **Solution:** Let us assume that one pair of shoes is worth $X$ and one pair of boots costs $Y$. Then, $Y/(4X) = 1.08$. Based on the obtained ratio, $Y/(5X) = Y/(4X) \cdot 4/5 = 0.864$. Thus, five pairs of shoes are 13.6% more expensive than one pair of boots.

**Answer:** 13.6%.

2.71. **Solution:** Let us assume that the distance from the tank farm to point A is equal to $X$. Then, the solution is based on the investigation of the function $f(X) = 80 \times 80 \times X + 70 \times 100 \times (40 - X)$ or $f(X) = -600X + 280000$. This is a linear function that decreases with an increase of $X$ from $X = 0$ km to $X = 40$ km, taking the minimum value at $X = 40$ km. Thus, the cost is minimal if the tank farm is built at the plant located at point B.

**Answer:** The tank has to be located at point B.
Chapter 11
Bar and Pie Graphs
Solutions and Answers

3.1. Answer:

Proportion of Total Daily Food

<table>
<thead>
<tr>
<th></th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Morning Breakfast</td>
<td>25%</td>
</tr>
<tr>
<td>Second Breakfast</td>
<td>15%</td>
</tr>
<tr>
<td>Lunch</td>
<td>45%</td>
</tr>
<tr>
<td>Dinner</td>
<td>15%</td>
</tr>
</tbody>
</table>

3.2. Answer:

Proportion of Total Daily Food

3.3. Answer:
3.4. **Answer:**

3.5. **Answer:** I.

3.6. **Answer:** IV.

3.7. **Answer:** Excellent—20 students; good—236 students; pass—80 students; fail—60 students.

3.8. **Solution:** According to the chart, the 60 students who did not pass this test comprise 15 percent of total students. The total number of students who took this test is 400. Therefore, 20 students received scores of excellent, 236 students scored good, and 80 students scored pass.

3.9. **Solution:** According to the graph, the number of the students who scored good and fail together is equal to 65%, and according to the problem condition it is 325 students. Therefore, the total number of students who took the test is 500, and the difference between good
and not passable scored people is 45%, or 225 students. Let’s assume
that X students received good scores and Y students failed. So, X +
Y = 325 and X – Y = 225. Solving two equations with two unknown
gives X = 275, Y = 50. According to the graph, 50 students, or 10%,
received excellent scores, and 125, or 25%, were satisfactory.

Answer: Fifty students scored excellent, 275 students scored
good, 125 students scored satisfactory, and 50 students failed.

3.10. Solution: The total number of sales is equal to 5,350 for Store #1
and 5,900 for Store #2.
Answer: Store #2.

3.11. Solution: The total amount of sales for the four months,
according to the chart, for Store #1 is 5,350 and for Store #2 is 5,900.
The income of Store #1 is $37,450, and the income of Store #2 is
$29,500.
Answer: Store #1.

3.12. Solution: The total number of sales in Store #2 is 6,000.
According to the problem, the number of sales in Store #1 should be
6000 ÷ 1.5 = 4000. Therefore, the number of March sales in Store #1
is equal to 700.
Answer: 700.

3.13. Solution: The total amount of sales in the first store is equal to A
+ A × (1 − 0.03) + A × (1 + 0.05) + A × (1 − 0.01) + A × (1 + 0.04)
= 1.05A, where A = the amount of sales in January. The total amount
of sales in the second store is equal to A + A × (1 − 0.03) + A × (1 −
0.03) × (1 + 0.05) + A × (1 − 0.03) × (1 + 0.05) × (1 − 0.01) + A × (1
− 0.03) × (1 + 0.05) × (1 + 0.04) = 1.056A.
Answer: The second store is more profitable after four months of
work.

3.14. Solution: It is seen from the graph that the law of the profit
growth can be expressed by geometric progression with the nth term
aₙ, with initial value a and common ratio r : aₙ = a × rⁿ⁻¹. In our
case, a = 1.5×1000, r = 2. Thus, the profit of the store during 12-th
year is 130,000 dollars. The sum of twelve numbers of geometric progression, which characterizes total profit after twelve years, is equal to $S_{12} = a \times \left( \frac{r^{12} - 1}{r - 1} \right) = 1.5 \times 10^3 \times \left( \frac{2^{12} - 1}{2 - 1} \right) = 6,142,500.$

Answer: The profit for 12-th year is $130,000 and for 12 years $6,142,500.

3.15. Answer:

3.16. Answer:

3.17. Answer: The average score of class A is equal to $\approx 70.79$ percent, and the average score of class B is $\approx 71.05$ percent. Class A has better test results than class B.
3.18. **Solution:** The total number of students who took the test was 130. According to the bar graph, the total number of students with excellent grades was 22 (≈ 16.92%), the number of students with good grades was 68 (≈ 52.3%), and the number of students with satisfactory grades was 40 (≈ 30.8%).

**Answer:** Excellent ≈ 16.92%, good ≈ 52.3%, satisfactory ≈ 30.8%.

3.19. **Answer:** 18 tickets were sold during the following days:

- (#1) + (#2) + (#6) + (#7)
- (#1) + (#5) + (#6) + (#7)
- (#2) + (#3) + (#4) + (#5) + (#6)
- (#4) + (#5) + (#6) + (#7).

3.20. **Solution:** For the first car, gasoline consumption (in liters per one hundred kilometers) in town and out of town together is $12 + 8 = 20$; for the second, $11 + 9 = 20$; for the third, $13 + 7 = 27$; and for the fourth, $14 + 8 = 22$. The results show that the first and second cars are the most efficient.

3.21. **Solution:** For the first car, gasoline consumption (in liters per one hundred kilometers) in town and out of town together is $12 + 2 \times 8 = 28$; for the second, $11 + 9 \times 2 = 29$; for the third, $9 + 2 \times 13 = 35$; and for the fourth, $8 + 2 \times 14 = 36$. The results show that the first car is the most efficient.

3.22. **Answer:**

\[
T_1 = \frac{-1 + 6 + 5 - 2 + 0 + 2}{6} \approx 1.7 \ C^0 \\
G_1 = \frac{(23.97 + 22.38 + 20.03 + 16.93 + 15.48 + 10.78)}{6} = 18.26 \ kWh \\
T_2 = \frac{(16 + 22 + 16 + 15 + 21 + 16)}{6} \approx 18 \ C^0
\]
Bar and Pie Graphs (Solutions and Answers)

\[ G_2 = \frac{(11.73 + 13.15 + 14.48 + 11.15 + 4.94 + 4.24)}{6} = 9.9 \text{ kWh} \]

3.23. Answer: Students of Ontario have an average of four years knowledge score equal to \(\approx 64\), Alberta \(\approx 70\), and Quebec \(\approx 78\). Thus, Quebec Province have an average of four years knowledge score higher than the students in Ontario and Alberta.

3.24. Answer:

3.25. Answer:
3.26. **Answer:** The highest revenue is in 4 stores for the month of March.

3.27. **Answer:** \( X = 6.7857 \times 7 - (5 + 5.3 + 5.4 + 7.3 + 7.6 + 9) \approx 7.9. \)

3.28. **Solution:** Let’s assume that the number of flyers delivered on Friday is \( X \) and the number of flyers distributed on Sunday is \( Y \). Then \( X + Y = 207 - 20 - 25 - 35 - 30 - 30 = 67 \). According to the statement of the problem \( X - Y = 3 \). \( X = 35 \), \( Y = 32 \).

**Answer:** There were thirty-five and thirty-two flyers distributed on Friday and Sunday, respectively.

3.29. **Solution:** Let’s assume that the number of flyers delivered on Friday is \( X \) and the number of flyers distributed on Sunday is \( Y \). Then \( X + Y = 209 - 20 - 25 - 35 - 30 - 30 = 69 \). According to the statement of the problem \( X / Y = 1.3 \). Therefore, \( X = 39 \), \( Y = 30 \).

**Answer:** There were 39 and 30 flyers distributed on Friday and Sunday, respectively.

3.30. **Solution:** To ensure good quality, the signal level received by a car antenna from all directions should be the same; therefore, the left diagram is preferable.

3.31. **Answer:** Diagram 1.

3.32. **Answer:** \( \phi = -90^0 \).

3.33. **Solution:**

a. Car #1 receives a signal that is greater than the signal received by Car #2 for azimuth incoming signal directions.

b. The signal received along the direction S for Car #2 is the strongest.

3.34. **Solution:** The average number of children \( \bar{N} \) per family is calculated as follows:
Bar and Pie Graphs (Solutions and Answers)

\[ \bar{N} = 0 \times 0.1 + 1 \times 0.3 + 2 \times 0.5 + 3 \times 0.1 = 1.6. \]

**Answer:** 1.6.

3.35. **Answer:**

3.36. **Answer:** Statement b is wrong.

3.37. **Answer:** Statement c is true.

3.38. **Answer:** Statement a is true.

3.39. **Answer:** Medium—250, small—125, large—125.

3.40. **Solution:** The true statement is b because the whole number of blue and red toys is more than a half of the total number of toys. They cannot only be large or small (the number of the large and small toys together is equal to the half of the total amount of toys).

3.41. **Solution:** There are one hundred people in the company, and the first diagram shows that all one hundred employees speak English. Therefore, seventy people who know only one language (second diagram) speak English.

**Answer:** 70.
### 3.42. **Solution:** According to the graphs, we can suggest the following combination of language knowledge:

a. Only English: 70 people  
b. Only French  
c. Only German  
d. English + French  
e. English + German: 2 people  
f. French + German  
g. English + German + French: 10 people

Based on these statements, we can conclude that the number of people who know English and French is eighteen because the number of people who know English, French, and German is ten, nobody knows just French and German, and the total number of those who speak French is 28.  
**Answer:** 18.

### 3.43. **Answer:** The number of medium-sized toys is exactly half of all toys—that is, 100. So, the number of green toys is 40, and therefore the number of yellow toys is 30.

### 3.44. **Solution:** Let’s assume that the number of white cars is X and the number of silver cars is Y. In accordance with the problem, $100 + X + 120 + 60 + Y = 520$ and $100 \times 60 = X \times Y$. The solution of the system of two equations with two unknowns is $Y = 150$ and $X = 90$.  
**Answer:** There are 90 white cars and 150 silver cars.

### 3.45. **Solution:** Let’s assume that the number of white cars is X and the number of silver cars is Y. In accordance with the problem, $100 + X + 120 + 60 + Y = 520$ and $X - Y = 60$. Therefore, $X = 150$ and $Y = 90$.  
**Answer:** There are 150 white cars and 90 silver cars.

### 3.46. **Solution:** Let’s assume that the number of white cars is X and the number of silver cars is Y. For the first condition, we have $(100 + X + 120 + 60 + Y) - (100 + X + 120) = 200$; for the second condition, we have $60 + Y = 100 + X$. Solving, we obtain $X = 100$ and $Y = 140$.  
**Answer:** There are 100 white cars and 60 silver cars.
3.47. **Answer**: Statement b is certainly not true.

3.48. **Answer**: Statement a may be true.

3.49. **Answer**: 21 seconds.

3.50. **Answer**: 81 seconds.
Chapter 12

Logic

Solutions and Answers

4.1. Answer: 5.


4.3. Answer: 4.5.

4.4. Answer: 5 days.

4.5. Solution: \((6/3) \times 2 = 4\).

4.6. Solution: No, because after 96 hours, it will be midnight again.

4.7. Answer: 1 hour and 30 minutes.

4.8. Solution: During a 24 hour period, the hour hand makes 2 revolutions, while minute hand goes around 24 times. Therefore, the minute hand overtakes hour hand 22 times. During one revolution, the minute hand forms a right angle with the hour hand twice.
Answer: 44.

4.9. Solution: Let us the number of lines wit the length of 7 cm is \(a\) and the number of the lines with the length of 12 cm is \(b\). Then \(4a + 6b = 28 + 72 = 100\), or \(a = 7\) and \(b = 12\).

4.10. Answer is shown in the figure below:
4.11. Solution:

Step #1: Two sons cross the river. Step #2: One of them returns. Step #3: The father crosses the river. Step #4: The second son returns. Step #5: The two sons cross the river.

4.12. Solution: It is necessary to tilt the container as shown in the figure below.

4.13. Solution: The solution is shown in the table below.

<table>
<thead>
<tr>
<th>Steps</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 liters</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>5 liters</td>
<td>5</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>5</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

4.14. Solution: Step #1: Fill the 11 liter pail with water. Step #2: Pour 6 liters of the water into the 6 liter container. Step #3: Dump half of the water from the 6 liter pail using the solution to problem #4.12. Step #4: Pour 3 liters from the 6 liter container into the pail containing 5 liters of water.
4.15. **Solution:** The solution requires nine steps as presented in the table below.

<table>
<thead>
<tr>
<th>Step</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 liters</td>
<td>8</td>
<td>5</td>
<td>5</td>
<td>2</td>
<td>2</td>
<td>7</td>
<td>7</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>3 liters</td>
<td>-</td>
<td>3</td>
<td>-</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>-</td>
<td>3</td>
<td>-</td>
</tr>
<tr>
<td>5 liters</td>
<td>-</td>
<td>-</td>
<td>3</td>
<td>3</td>
<td>5</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

4.16. **Solution:** Let us label the 19 liter container A, the 13 liter container B, and the 7 liter container C.

1) Pour all the water from C into A and pour the contents of B into A until full. You should have 19 liters in A and 1 liter in B (19, 1, 0);  
2) Pour A into C. Result: (A->12 liters, B->1 liter, C->7 liters);  
3) Pour C into B. Result: (A->12 liters, B->8 liter, C->0 liters);  
4) Pour A into C. Result: (A->5 liters, B->8 liter, C->7 liters);  
5) Pour C into B. Result: (A->5 liters, B->13 liter, C->2 liters);  
6) Pour B into A. Result: (A->18 liters, B->0 liter, C->2 liters);  
7) Pour C into B. Result: (A->18 liters, B->2 liter, C->0 liters);  
8) Pour A into C. Result: (A->11 liters, B->2 liter, C->7 liters);  
9) Pour C into B. Result: (A->11 liters, B->9 liter, C->0 liters);  
10) Pour A into C. Result: (A->4 liters, B->9 liter, C->7 liters);  
11) Pour C into B. Result: (A->4 liters, B->13 liter, C->3 liters);  
12) Pour B into A. Result: (A->17 liters, B->0 liter, C->3 liters);  
13) Pour C into B. Result: (A->17 liters, B->3 liter, C->0 liters);  
14) Pour A into C. Result: (A->10 liters, B->3 liter, C->7 liters);  
15) Pour C into B. Result: (A->10 liters, B->10 liter, C->0 liters);

4.17. **Solution:** The solution is presented in the six steps in the table below.

<table>
<thead>
<tr>
<th>Step</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>bottle 6 l</td>
<td>6</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>bottle 5 l</td>
<td>0</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Bottle 1 l</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
4.18. **Answer:** $99+9=108$.


4.20. **Answer:** $993+58=1051$.

4.21. **Answer:** Irina and Yelena used currant, while Tania and Olga used gooseberry.

4.22. **Solution:** The girl in green is Gail because according to the problem, the girl in green is not Anna, not Vicky, and not Nadia. The girl in white is not Vicky or Nadia, because Nadia is near to the green girl and white girl is close to Vicky. Thus, the girl in white is Anna. According to the problem, Vicky is not in pink. Thus, Vicky is in blue, and therefore Nadia is in pink.

   **Answer:** Anna is in white; Vicky is in blue; Gail is in green; and Nadia is in pink.

4.23. **Solution:** The spoon first removes 10 units of milk, so the milk cup contains 90 grams of milk, and the coffee cup contains 100 grams of coffee and 10 grams of milk. After mixing the coffee and milk in the cup of coffee, each gram of the mixture contains $10/11$ of coffee and $1/11$ of milk. With a spoon, we take out 10 grams of the mixture and pour it into the cup with milk. Because one spoon contains 10 grams of mixture, we transfer $100/11$ of coffee and $10/11$ grams of milk into the cup of milk. Finally, the milk glass contains $90+100/11=1000/11$ grams of milk and $100/11$ grams of coffee.

   **Answer:** $100/11$ grams of coffee.

4.24. **Solution:** The spoon first removes 10 units of milk, so the milk cup contains 90 grams of milk, and the coffee cup contains 100 of coffee and 10 grams of milk. After mixing the coffee and milk in the cup of coffee, each gram of the mixture contains $10/11$ of coffee and $1/11$ of milk. Because one spoon contains 10 grams of the mixture, we transfer $100/11$ of coffee plus $10/11$ grams of milk into the glass of milk. So, finally, the milk cup contains $90+100/11=1000/11$ grams of milk and $100/11$ grams of coffee. The coffee cup contains 100-
100/11 = 1000/11 grams of coffee and 10-10/11 = 100/11 grams of milk. Therefore, equal amounts of milk and coffee were transferred.

**Answer:** Equal amounts of milk and coffee were transferred.

4.25. **Answer:** The weight of the iron and fluff is the same.

4.26. **Answer:** A cup of coffee is worth $1.05 and a spoon of sugar costs $0.05.

4.27. **Answer:** One man has to take the basket with the apple in it.

4.28. **Solution:** One liter of milk weighs about 1 kg. The packaging with $3 \times 3 \times 3$ liters weighs 27 kg, while the weight of the $9 \times 9 \times 9$ package is equal to 729 kg. Because $729/3 \gg 27$, three workers will not be able to lift the $9 \times 9 \times 9$ package.

**Answer:** No.

4.29. **Solution:** Let’s assume that there were originally $A$ candies in the basket. Obviously, the basket finally had $A/2/2/2 = 10$. From this equation, we can obtain $A = 160$ candies.

**Answer:** 160 candies.

4.30. **Solution:** After the first strike, we have three parts; after the second, five parts; after the third, seven parts; and so on. Hence, after $N$ strikes, we have $2N+1$ parts. Thus, $2N+1=2007$ or $N=1003$.

**Answer:** 1003.

4.31. **Solution:** It is necessary to pull out three socks. Possible combinations are as follows:

a. All three socks are black,
b. All three socks are white,
c. One sock is white, two socks are black,
d. One sock is black, two socks are white.

**Answer:** Three socks.
4.32. **Solution:** Let’s assume that for the first seller, the number of 2 kg bags is $X_1$, and for the second seller, this number is $Y_1$. The number of 3 kg bags for the first seller is $X_2$, and that for second seller is $Y_2$. Determine the following combinations of $X_1$ and $X_2$ to get $3X_1+2X_2=15$:

- $X_1=1$, $X_2=6$
- $X_1=3$, $X_2=3$
- $X_1=5$, $X_2=0$

Determine the following combinations of $Y_1$ and $Y_2$ to get $3Y_1+2Y_2=20$:

- a. $Y_1=0$, $Y_2=10$
- b. $Y_1=2$, $Y_2=7$
- c. $Y_1=4$, $Y_2=4$
- d. $Y_1=6$, $Y_2=1$

Comparing the results, we conclude that the first and second sellers can divide their sacks into seven small bags as follows:

The first one will have one 3 kg bag and six 2 kg bags; the second seller will have six 3 kg bags and one 2 kg bag.

4.33. **Answer:** 5+2+7 students are in the class.

4.34. **Solution:** Let’s assume that half of the total number of bags is $X$. In addition, we will assume that Mila used $Y$ bags to make two cups of tea per bag and $X-Y$ bags to make three cups of tea per bag. Tania used $Z$ bags to make two cups per bag and $X-Z$ bags to make three cups per bag. Then, according to the problem,

$$2Y+3(X-Y)=3X-Y=23 \ (1) \quad \text{and} \quad 2Z+3(X-Z)=3X-Z=32 \ (2).$$

Let’s sum the two equations:

$$6X-(Y+Z)=55 \ (3).$$

From equation (3), it follows that $X$ has to be equal to or more than 10, and from equation 3 it follows that $X$ has to be more than 10;
otherwise, \((Y+Z)\) or \(Z\) is less than 0. In addition, according to our notations,
\[
X>Y \quad (4) \quad \text{and} \quad X>Z \quad (5).
\]
The solution to equation (1) and inequality (4) gives \(X<(23/2)\), or \(X<12\). Therefore \(X=11\), \(Y=10\) and \(Z=1\).
The total number of bags is \(2X=22\).
\text{Answer: 22.}

4.35. \textbf{Solution:} The false coin can be determined after four weighings. The algorithm is as follows. First weighing: Put 27 coins on each pan of the balance scale. In the case of a correct balance, the false coin is among the 26 remaining coins. In the case of incorrect balance, we find 27 coins with one false one. Choose these 27 coins and divide this amount into three parts. Second weighing: Put nine coins on one side of the scale and nine coins on the other. Thus, we can find the group of nine coins with a false one. Third weighing: Divide the nine coins with one false in three parts and weigh three coins against three others. Final weighing: Take three coins with one false; chose any two, weigh them, and determine which is false. If the false coin is among the 26 remaining coins which we did not weigh at the beginning, we add to the 26 coins one good one chosen from the 27 good coins and repeat our procedure.
\text{Answer: Four weighings.}

4.36. \textbf{Solution:} Put a pile of 50 coins on the left side and 50 coins on the right side. Two cases are possible:

Case #1: Equal weight of the left and right piles. Take the rest of the coins and put them in the left pile instead of one which is normal:

a) The left pile of coins is heavier => the false coin is heavier;

b) The left pile of coins is lighter => the false coin is lighter.

Case #2: The left and right piles are of unequal weight. Take the heavier pile and divide it into two piles with 25 coins each:

a) The left and right weigh the same => the false coin is lighter;

b) The left and right weigh the same => the false coin is heavier.
4.37.  **Solution:** Let’s show the possible options, as follows:

- Serge: C, Victor: B, Alex: A; or
- Serge: B, Victor: C, Alex: A; or
- Serge: A, Victor: B, Alex: C.

The teacher was wrong twice; therefore, the correct answer is:

Serge: C, Victor: B, Alex: A.

**Answer:** Serge: C, Victor: B, Alex: A.

4.38.  **Solution:** Since a girl goes to kindergarten, Boris is not 5 years old. Because Anna is older than Boris, Anna must be 13 or 15 years. But the sum of years of Anna and Vera is divisible by 3, so Anna 13 years, while Vera 5 years old. Because Anna is older than Boris, Boris is 8 years. Finally, Gail is 15 years.

**Answer:** Vera is 5 years old, Boris is 8, Anna is 13, and Gail is 15.

4.39.  **Solution:** Let us assume that the number of boys is $x$. Then, \( \frac{60}{x} - \frac{60}{35-x} = 1 \). Solving the quadratic equation, we get $x=15$ (number of boys); $35-x = 20$ (girls).

**Answer:** 15 boys and 20 girls.

4.40.  **Solution:** It is known that the delay is equal to 8 minutes for 4 full days. This means that one day (24 hours) corresponds to a 2 minutes delay. There are 3.5 days or 84 hours between 8 pm Saturday and 8 am Wednesday. Therefore, the alarm clock will be 7 minutes behind.

**Answer:** 7 minutes.

4.41.  **Solution:** Let’s denote the amount of fish caught by each boy by the first letter of his name and write the conditions of the problem algebraically. We obtain three statements, as follows:

1) $V>D$; 2) $S+P=V+D$; 3) $V+S<D+P$.

Adding the term by term 2 and 3, we obtain $S+P+V+S<V+D+D+P$ or $S<D$. Next, using the obtained result and condition 2, we obtain $P>V$. Taking into account inequality 1, this can be written $P>V>D>S$.

**Answer:** Philip caught the most fish and Shawn caught the least.
4.42. **Solution:** Let’s assume that the number of desks that are occupied by girls is equal to $N$. Then, the number of girls that are sitting with girls is $2N$. The number of boys sitting with boys is $4N$. The number of girls sitting with boys is $4N$; the number of boys sitting with girls is the same. Thus, the total number of boys is $8N$, and the number of girls is $6N$. Therefore the boy/girl ratio is equal to $8/6=4/3$.

   **Answer:** $4/3$.

4.43. **Solution:** One of the possible solutions is as follows: He passed eight exams at the end of the fourth year, nine at the end of the fifth year, three at the end of the first year, four at the end of the second year, and seven at the end of the third year.

4.44. **Solution:** If three Wednesdays occurred on even numbers in one month, then the first Wednesday is the 2nd of the month, the third is the 16th, and the fifth is the 30th. Therefore, the second Sunday is the 13th of the month.

4.45. **Solution:** In the worst case, the appropriate suitcase for the first key will be found in four attempts, the second in three attempts, the third in two, the fourth in one, and the fifth key can only fit the remaining suitcase. Therefore, in the worst case, 10 attempts are required.

   **Answer:** 10 attempts.

4.46. **Solution:** Since one year contains 12 months, and there are 37 students in the group, there must be a month of the year in which more than 3 students celebrate birthday.

4.47. **Solution:** The total number of blue and green pencils can be divided by 7. Thus, the total number of blue and green pencils together is equal to 7 or 14. The number of red pencils in the first case is 13, which is more than the number of blue pencils. However, according to the problem, this number has to be less than the number of blue pencils. Therefore, the solution in the first case would be 13
red pencils, which is more than the number of blue ones. Hence, the second option is the only possible one: There are six red pencils.

**Answer:** 6 red pencils

**4.48. Solution:** Let’s assume that the age of the youngest brother is $X$. Then, the age of the middle brother is $X+2$ and that of the oldest brother is equal to $2X+2+4$. The total sum is 96. Therefore, $X=22$, $X+2=24$, and $2X+6=54$.

**Answer:** The youngest brother is 22 years old, the middle one is 24, and oldest is 54.

**4.49. Solution:** Let $K$ be Irina’s apartment number (and Leon’s floor number). Then, Leon’s apartment number must be between $9K-8$ and $9K$; hence, the number of apartments is from $10K-8$ to $10K$. The system of inequalities $10K-8 \leq 329 \leq 10K$ has a unique solution: $K=33$ and Leon lives in an apartment with the number $329-33=296$.

**Answer:** 296.

**4.50. Solution:** The number of tigers was seven times greater than that of the other animals. So, this number is $7/8$ of the total number of animals. Since the number of monkeys is 7 times less than the other animals, the number of monkeys $1/8$ of the total animals amount. Because $7/8 + 1/8 = 1$, there were no lions at the zoo.

**4.51. Solution:** Let’s assume that the number of positive target hits is equal to $n$. Then, the condition of the problem can be written as an inequality $5n-2(10-n) \geq 30$, or $7n \geq 50$, or $n \geq 7$.

**Answer:** $n=8$.

**4.52. Solution:** Let’s assume that a book has $N$ pages. Then, the reading time with speed $V_1$ is $T_1=N/V_1$, in the second, $T_2=N/V_2$. $T_2-T_1=N(1/V_2-1/V_1)=N(V_1-V_2)/(V_1V_2)=120$ minutes. When $V_1=1$ page per minute, and $V_2=0.5$ pages per minute, we obtain $N=120$ pages.

**Answer:** 120 pages.
4.53. **Solution:** Let’s assume that first tank contains $X$ liters of water, the second tank contains $Y$ liters, and third tank contains $Z$ liters. According to the problem, we can write the following:

\[
\begin{align*}
X + Y + Z &= 240 \\
3(X - 48) &= Z \\
3(Y - 72) &= Z
\end{align*}
\]

The solution of the system gives: $X=72$ liters, $Y=96$ liters, $Z=72$ liter.

**Answer:** First tank contains 72 l, second – 96 l, and third – 72 l.

4.54. **Solution:** Let’s assume that the number of cents in the first pocket is $X$. Then, the second friend has $X+A$ cents, where $A$=common difference, and the third friend has $X+2A$ cents. According to the problem,

\[X+X+A+X+2A=3X+3A=30\]

On the other hand, $X$, $X+A-2$, and $X+2A$ are terms of a geometric sequence. In this case,

\[X+A-2=X\times q \quad (2)\]
\[X+2A=X\times q^2 \quad (3)\]

Thus, we have three equations with three unknowns, namely $X$, $A$, and $q$. The solution of this system is as follows: $X=4$, $A=6$, $q=2$ and $X+A=10$, $X+2A=16$.

**Answer:** 4; 10; 16.

4.55. **Answer:** Vladimir’s parents have one more daughter than they have sons.

4.56. **Answer:** Five people and two dogs.

4.57. **Solution:** As follows from the problem, among 23 students, 17 are taller than Victor and 13 are shorter than Peter. Therefore, Victor is shorter than Peter. Because 17 students are taller than Victor and the total student number is 23, the number of people who are shorter
than Victor is equal to 5 \((17+5+\text{Victor}=23)\). Thus, the number of people who are shorter than Peter and taller than Victor is \(13-5-\text{Victor}=7\).

**Answer:** 7.

4.58. **Solution:** It transported \((10+10+10)\) cars plus \((6+6)\) trucks.

**Answer:** 12.

4.59. **Solution:** Yes, you can, if you ship two containers weighing 170 kg and 14 weighing 190 kg, for a total of 3000 kg.

**Answer:** Yes

4.60. **Solution:** Six possible age options among the boys are presented in the table below.

<table>
<thead>
<tr>
<th>Oldest</th>
<th>Middle</th>
<th>Youngest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cohen</td>
<td>Boris</td>
<td>Alex</td>
</tr>
<tr>
<td>Cohen</td>
<td>Alex</td>
<td>Boris</td>
</tr>
<tr>
<td>Boris</td>
<td>Cohen</td>
<td>Alex</td>
</tr>
<tr>
<td>Boris</td>
<td>Alex</td>
<td>Cohen</td>
</tr>
<tr>
<td>Alex</td>
<td>Boris</td>
<td>Cohen</td>
</tr>
<tr>
<td>Alex</td>
<td>Cohen</td>
<td>Boris</td>
</tr>
</tbody>
</table>

If statement 1 is true, then conditions “a” and “b” are false. However, only one condition among a, b, c and d is false. Therefore, statement 1 is false.

If statement 2 is true, then “b” and “d” are false. Thus, statement 2 is false.

If statement 3 is true, then “a” and “b” are false. Thus, statement 3 is false.

If statement 4 is true, then “a” and “c” are false. Thus, statement 4 is false.

If statement 5 is true, then “c” and “d” are false. Thus, statement 5 is false.

If statement 6 is true, then only “b” is false. Thus, statement 6 is true.
Answer: Boris is the youngest.

4.61. Solution: The total number of parliament deputies is even (there is the same number of deputies in both houses). Consequently, the total number of deputies who voted for and against the decision will also be even. Thus, the difference between the number of votes “for” and “against” should also be even. Therefore, the advantage (i.e., the difference between the number of people voting for the decision and the number of voting against it) cannot be equal to 23. This statement is false.

4.62. Solution: First, we should place the boys in the queue as specified in the problem: Yuri, Michael, Vladimir, Scott, and Otto. As we can see, this is the wrong sequence, as condition #2 is not satisfied. Let’s adhere to condition #2, and we will have the following line: Otto, Yuri, Michael, Vladimir, Scott. However, this line does not adhere to condition #2. So, we replace this with the following queue: Otto, Yuri, Vladimir, Michael, Scott. This satisfies all of the problem conditions.

Answer: Otto, Yuri, Vladimir, Michael, Scott.

4.63. Solution: Let’s find the number of kids who attend only drama club and nothing else. According to the problem, this number is 27-10-3-8=6. The number who only sing in the choir is 32-10-3-6=13. There are 22-8-3-6=5 who are only sportmen. Therefore, the number of teenagers who don’t attend any of above described groups is equal to 70-6-13-5-8-10-3-6=19.

Answer: 19.
Chapter 13

Linear and Circular Motion

Solutions and Answers

5.1. Answer: 1.2 hours.

5.2. Answer: 80 km/h.

5.3. Solution: The speed of 60 km/h corresponds with the rate of one kilometer per minute. A first wagon of a train travels the tunnel end in one minute and last wagon in the other minute. The total time is 2 minutes.

Answer: 2 min.

5.4. Solution: It is known that the relationship between distance $S$, speed $V$, and time $T$ is found in the formula

$$\frac{S}{V} = T \quad (1)$$

When speed increases by 20 percent, (speed increases by 1.2 times), moving time becomes equal to $T - 1$. The relationship between distance $S$, speed $1.2V$, and time $T - 1$ is found from the following formula:

$$\frac{S}{1.2V} = T - 1 \quad (2)$$

System of two linear equations
Motion (Solutions and Answers)

\[
\begin{align*}
\frac{S}{V} &= T \\
\frac{S}{1.2V} &= T - 1
\end{align*}
\]

gives the following solution: \( T = 6 \). Note that solution does not depend on the distance value \( S \).

**Answer:** 6 hours.

5.5. **Solution:** Let’s assume that train has to pass distance \( S \). The total time of the train movement is as follows:

\[
\frac{S}{2V} + 2 + \frac{S}{2(V + 10)} = \frac{S}{V} \quad \text{or} \quad 10 \times S = 4 \times V \times (V + 10) \quad (1)
\]

The equation to find \( V \) can be found from the last condition problem:

\[
\frac{S}{V - 10} - \frac{S}{V} = 6 \quad \text{or} \quad 10 \times S = 6 \times V \times (V - 10) \quad (2)
\]

System of two equations (1) and (2) with two unknowns \( S \) and \( V \) gives solution for \( S \): \( S = 1,200 \) km.

**Answer:** 1,200 km.

5.6. **Solution:** Let’s assume that train has to pass distance \( S \). The total time of the train movement is as following

\[
\frac{S}{2V} + 2 + \frac{S}{2(V + 10)} = \frac{S}{V} \quad \text{or} \quad 10 \times S = 4 \times V \times (V + 10) \quad (1)
\]

The equation to find \( V \) can be found from the last condition problem:

\[
\frac{S}{V - 10} - \frac{S}{V} = 6 \quad \text{or} \quad 10 \times S = 6 \times V \times (V - 10) \quad (2)
\]

System of two equations (1) and (2) with two unknowns \( S \) and \( V \) gives solution for \( V \): \( V = 50 \) km/h.

**Answer:** 50 km/h.
5.7. **Solution**: Because the first part of the way before lunch (10 km) is equal to 50% of the entire route the entire route is equal 20 km. Let’s assume that initial speed rate is $V$. After lunch during the moving his speed was equal to $(V + 1)$ km/h. The time of moving before lunch $\frac{10}{V}$, after lunch $\frac{10}{V + 1}$. The total time is equal to $T = \frac{10}{V + 1} + \frac{10}{V + 1}$. This relationship is equal to $20 / V$, because he arrived to destination without delay. So,

$$\frac{20}{V} = \frac{10}{V} + \frac{10}{2} + \frac{10}{V + 1}.$$ This equation is an equivalent to the following: $V^2 + V - 20 = 0$. Positive root solution is $V = 4$.

**Answer**: The initial speed rate is 4 km/h.

5.8. **Solution**: The time it takes for the airplane to travel from point A to point B is $T_1 = \frac{S}{V + U}$, and the time it takes for the airplane to fly back is $T_2 = \frac{S}{V - U}$, where $S =$ distance from A and B, $V =$ speed of the airplane without any wind, and $U =$ speed of the wind. The ratio is $\frac{T_2}{T_1} = \frac{V + U}{V - U}$, which equals $13 ÷ 12 ≈ 1.1$.

**Answer**: $≈ 1.1$.

5.9. **Solution**: Let’s assume that $V_1 =$ speed of the boat in still water and the rate of a still water is $V_0$.

Time $T_1 = \frac{S}{V_1 + V_0} + \frac{S}{V_1 - V_0} = \frac{2S \times V_1}{V_1^2 - V_0^2}$, $T_2 = \frac{2S}{V_1}$. The ratio $T_1 = \frac{V_1^2}{V_1^2 - V_0^2} > 1$.

**Answer**: $T_1 > T_2$.

5.10. **Solution**: Let’s say that the tourists covered $X$ km the first day. The second day, they covered $(X + 10)$ km; the third day, they covered $(X - 5)$ km; and the fourth day, they covered $X$ km. So,
they traveled \((4X + 15) = 95\) km in all four days. The equation \((4X + 15) = 95\) gives a solution \(X = 20\) km.

**Answer:** Tourists covered 20 km on the first day, 30 km on the second day, 25 km on the third day, and 20 km on the fourth day.

5.11. **Solution:** Let’s say \(X\) km/h is a rate of the motorboat in still water, and \(Y\) km/h is the speed of still water. Moving upstream, the boat traveled \(1 / 4 \times (X - Y)\) km from a bridge in fifteen minutes when the boatman discovered that he had lost the bottle. He turned back and traveled \(1 / 4 \times (X - Y) + 2\) km at a rate \(X + Y\) km/h in time

\[
T_1 = \frac{\left(\frac{1}{4} \times (X - Y) + 2\right)}{(X + Y)} \quad (1).
\]

The bottle runs two kilometers at a speed \(Y\) in time

\[
T_2 = \frac{2}{Y} \quad (2)
\]

\[
T_2 - T_1 = \frac{1}{4} \quad (3)
\]

Solving equations (1), (2), and (3) gives \(Y = 4\) km/h.

**Answer:** 4 km/h.

5.12. **Solution:** Let’s assume that the escalator length is \(L\). The woman’s speed is equal to \(V_0\), and the speed of the escalator is \(V_1\). Let’s \(T\) is a time to up or down for one escalator step. The time to walk down from the top to the bottom is

\[
\frac{L}{V_0 + V_1} = T \times n_1 = 100 \times T \quad (1)
\]

The time to walk up from the bottom to the top is

\[
\frac{L}{V_0 - V_1} = T \times n_2 = 300 \times T \quad (2)
\]

The time to walk from the top to the bottom or from the bottom to the top if an escalator does not move is
\[ \frac{L}{V_0} = T \times n \]. Equation (1) and (2) give \( V_0 = 2V_1 \). Therefore, \( n = 150 \).

Answer: \( n = 150 \).

5.13. Solution: Triangles ABD and ECD are similar. Therefore, \( \frac{AB}{EC} = \frac{AD}{ED} \). Let’s say \( AD = X \). Then \( 10 = (4 + X) \times X \). \( X = 1 \)

Answer: The length of the shadow ED is one meter.

5.14. Solution: The initial shadow length AD as it follows from the previous problem solution is equal to five meters. When the distance between the person and the light post is equal to \( (4 + 6) = 10 \) meters a length ED is 2.5 meters (it follows from similar triangles ABD and ECD). So, the answer is \( (10 + 2.5 - 5) = 7.5 \) meters.

Answer: 7.5 m.
5.15. **Solution**: The initial distance between the man and the light pole is equal to $L_0$. For this distance $(AD)_0 = \frac{L_0 \times H}{H-h}$. The new man’s position $L_1$ gives $(AD)_1 = \frac{L_1 \times H}{H-h}$. Let’s denote $\Delta_1 = L_1 - L_0$ and $\Delta_2 = (AD)_1 - (AD)_0 = \frac{H}{H-h} \times (L_1 - L_0)$. A ratio $q = \frac{V_2}{V_1}$ between the speed of a shadow and a speed of the man with respect to the light pole is equal to $q = \frac{V_2}{V_1} = \frac{\Delta_2}{\Delta_1} = \frac{H}{H-h}$ (the moving time for man and shadow is the same).

**Answer**: $q = \frac{V_2}{V_1} = \frac{\Delta_2}{\Delta_1} = \frac{H}{H-h}$.

5.16. **Solution**: Let’s estimate the position of point D in the figure below as a function of the man’s position. Let’s say that the initial distance between the man and the pole is equal to $L_0$ and the new
distance is $L_1$. Denote $\Delta_1 = L_1 - L_0$. The initial shadow length is 

$$(ED)_0 = \frac{h \times L_0}{H-h},$$

and the new length of the shadow is 

$$(ED)_1 = \frac{h \times L_1}{H-h}. \quad \Delta_2 = (ED)_1 - (ED)_0 = \frac{h}{H-h} \times (L_1 - L_0)$$

with respect to a man. A ratio $q = \frac{V_2}{V_1}$ between speed of a shadow and a speed of a man is equal to 

$$q = \frac{V_2}{V_1} = \frac{\Delta_2}{\Delta_1} = \frac{h}{H-h}$$

(moving time for man and shadow is the same). Let’s examine three different cases:

a. $\frac{\Delta_2}{\Delta_1} < 1$ or $h < \frac{H}{2}$. The shadow moves at a speed slower than the man himself with respect to a light pole.

b. $\frac{\Delta_2}{\Delta_1} > 1$, or $h > \frac{H}{2}$ and according to the math problem $h < H$. The shadow moves faster than the man with respect to a light pole.

c. $\frac{\Delta_2}{\Delta_1} = 1$ or $h = \frac{H}{2}$. The speed of the shadow with respect to the man is equal to the speed of the man with respect to the light pole.
5.17. Solution: The average speed is determined by dividing traveled distance over time. The time it takes to travel the first half of the trip is 72 minutes. The time it takes to travel the second part of the trip is 90 minutes. The total time it takes to cover 240 km is equal to 162 minutes. So, the average speed is $240 / (162 / 60) \approx 88.9 \text{ km/h}$.
Answer: $\approx 88.9 \text{ km/h}$.

5.18. Solution: The average speed is determined by dividing traveled distance over time. The time it takes to travel the first half of the trip is 72 minutes. The time it takes to travel the second part of the trip is 80 minutes. The total time it takes to travel the full distance including the rest time is 192 minutes. Then, the average speed during the trip is equal to $240 / (192 / 60) = 75 \text{ km/h}$.
Answer: 75 km/h.

5.19. Solution: Let’s say the distance between the start and end of the trip is equal to $S$. The time it takes to travel the first third of the trip ($S / 3$) with 100 km/h is equal to $S / 300$. Rest time, which is a third of the first part of the trip, is equal to $S / 900$. The time it takes to travel the last part of the trip, which is two-thirds of the total distance, is equal to $2S / (3 \times 120) = S / 180$. The total time $T$ it takes to travel the full distance is equal to $T_{\text{total}} = (S / 300 + S / 900) + S / 180 = S / 100$. Therefore, the average speed $V$ is found as $S / T_{\text{total}} = 100 \text{ km/h}$.
Answer: 100 km/h.

5.20. Solution: Let’s assume the route distance is $S$. The first part of the route, the driver was moving at a speed $t_1 = S / (2V_1)$. The second part of the way, he drove at a rate $V_2 = S / 2V_2$. The average speed is $V_{av} = \frac{S}{2V_1 + \frac{S}{V_1 + V_2}} = \frac{2V_1 V_2}{V_1 + V_2} = 42 \text{ km/h}$. Because $V_{av} < V_0$, the driver does not get to the airport in time.

5.21. Solution: Let’s say a distance between City A and B is equal to
S. A boat travels $S$ km downstream in a time of $T_1 = \frac{S}{V_1 + V_0}$ and upstream in a time of $T_2 = \frac{S}{V_1 - V_0}$. Total moving time is equal to $T = T_1 + T_2$. The average speed is

$$V_{av} = \frac{2S}{T} = \frac{2S}{\frac{S}{V_1 - V_0} + \frac{S}{V_1 + V_0}} = \frac{V_1^2 - V_0^2}{V_1}.$$

Answer: $V_{av} = \frac{V_1^2 - V_0^2}{V_1}$.

5.22. **Solution:** The car travels for 60 km at a rate of 90 km/h in 40 minutes in the inner lane, 35 km at a speed of 105 km/h in 20 minutes in the central lane, and 120 km in an outer lane in one hour. So, the car covers 215 km in two hours. Therefore, the average speed is equal to 107.5 km/h.

Answer: 107.5 km/h.

5.23. **Answer:** 50 km/h.

5.24. **Solution:** A bus covers the distance of 90 km in $90 / 60 = 1.5$ hours. A car has only one hour to overtake a bus because it leaves City A after half an hour. Driving at a rate of 80 km/h, the car travels 80 km in one hour, which is less than the distance between A and B. Therefore, the car does not overtake the bus on the way from A to B.

5.25. **Solution:** Let’s assume that a cyclist travels at a speed of $V_1$ and, before meeting, covered distance $S_1$ in time $T_1$. A car’s driver traveled at a speed of $V_2$ before meeting covered distance $S_2$. According to the problem, the cyclist covered a distance $S_2$ in four hours. So, the total moving time for a cyclist is $T = S_1 / V_1 + 4$, for car driver $S_2 / V_2 = T_1$, $S_1 / V_2 = 1$. To get a solution, the following equations have to be satisfied:
\[
\frac{S_1}{V_1} = T_1 \quad \frac{S_2}{V_1} = 4 \quad \frac{S_2}{V_2} = T_1 \quad \frac{S_1}{V_2} = 1
\]

So, these fur equations: \( V_2 = 2V_1 \), and \( S_1 \div V_1 = 2 \), Therefore, \( T = S_1 \div V_1 + 4 = 6 \).

Answer: 6 hours.


5.27. Solution: Let’s assume that the first train left a location at 6:00 a.m. at a speed of \( X \) km/h, and the second train left the location at 6:30 a.m. at a speed of \( (X + 15) \) km/h. Therefore, the first train covers a distance equal to \( X \) km in one hour (from 6:00 a.m. to 7:00 a.m.), and the second train covers a distance of 0.5\((X + 15)\) km in half an hour (from 6:30 a.m. to 7:00 a.m.). At 7:00 a.m., they are 135 km apart. So, \( X + 0.5 \times (X + 15) = 135 \). Therefore, the speed of the second train is 100 km/h.

Answer: 100 km/h.

5.28. Solution: The rate of the first cyclist is 15 km/h, and the rate of the second cyclist is 20 km/h. So the distance traveled by the first in time \( T \) is 15\( T \), and the distance traveled by second cyclist in time \( T \) is 20\( T \). The distance traveled by the first and second cyclists together gives us an equation that can be solved for \( T \):

\[15T + 20T = 635, \text{ or } T = 4 \text{ hours.}\]

Answer: 4 hours.

5.29. Solution: Let’s assume that the average speed of the bus is \( X \). Then, the average speed of the car is equal to \( 2X – 30 \). The distance \( S \) between car and bus in two hours of traveling is \( S = 4X – 60 – 2X \). Using the fact that \( S = 60 \) km, the equation \( 4X – 60 – 2X = 60 \) gives \( X = 60 \) km, and the average speed of the car is \( 2X – 30 = 90 \) km/h.

Answer: 90 km/h.

5.30. Solution: The time delay at the end of the loop between slow and fast runners is equal to \( T_1 – T_2 = 5 \) seconds. However, the time delay between start of runners is \( \Delta t = 2 \) sec. Therefore, fast runner will meet slow runner at the first loop. The slow runner covers
distance $D_1 = \frac{(t_x + \Delta t)l}{T_1}$ before he meets the fast runner in time $t_x$.

$l = \text{the length of the loop, } \frac{l}{T_1} = \text{speed of the slow runner. The fast}$

runner covers $D_2 = \frac{t_x l}{T_2} = \text{distance before he meets the slow runner},$

$\frac{l}{T_2} = \text{speed of the fast runner. Relationship } D_1 = D_2 \text{ gives}$

$(t_x + \Delta t)l = t_x l. \text{ So, } t_x = \frac{\Delta t T_2}{T_1 - T_2} = 26 \text{ seconds.}$

**Answer:** 26 sec.

**5.31. Solution:** Since the traffic density of cars in direction 2 is 3.5 times greater than direction 1, the green light in direction 2 must be on 3.5 longer than that of direction 1. $2 \times 3.5 = 7 \text{ minutes.}$

**Answer:** 7 min.

**5.32. Answer:** 250 m.
5.33. **Answer:** 250 m.

5.34. **Answer:** 90 km/h.

5.35. **Answer:** 24 sec.

5.36. **Answer:** 6 hours.

5.37. **Answer:** 110 km/h.

5.38. **Answer:** 120 km/h.

5.39. **Answer:** 123 km/h.

5.40. **Solution:** By the first stage of the route, the cyclist traveled 30 km north. During the second part of the trip, he covered 40 km east, and during the third stage, he ran 50 km southwest. Values 30, 40, and 50 give three Pythagoras values. So, Graph A below shows how far the cyclist moves.

![Graph A](image)

5.41. **Solution:** The time to cross a river \( t \) is equal to \( t = \frac{S_1}{V_1} \). Therefore, the distance \( S_2 \) the man drifts along the water stream is \( S_2 = V_2 \times t = V_2 \times \frac{S_1}{V_1} = 0.1 \times \frac{100}{0.25} = 40 \) m.

![Diagram](image)
Answer: 40 m.

5.42. Solution: In one hour, the second cyclist will ride 10 km in distance AB in the figure below, and the first cyclist will ride AC = 20 km, which is double the distance of the second cyclist (refer to the diagram for a visual of their paths). Since the distance of the second cyclist is half of that of the first cyclist and their paths are $60^\circ$ apart, angle ABD=30°. $BC = \sqrt{DC^2 + AB^2 - AD^2}$, where $AD = AB / 2 = 5$ (as a side opposite the $30^\circ$ in right triangle), $DC = AC - AD = 15$. Therefore, $BC \approx 17.3$ km.

Answer: $\approx 17.3$ km.

5.43. Solution: Each minute, any point of a wheel travels a distance equal to $S = 2\pi n$. The linear bicycle speed is

$$V = \frac{S}{T} = \frac{2\pi R n}{T} = \frac{2 \cdot 3.14 \cdot 0.4 \cdot 100}{60} = 4.19 \text{ m/sec} \approx 15 \text{ km/h}.$$
5.44.  **Answer:** $2\pi R N$.

5.45.  **Answer:** $N = \frac{4800}{0.75 \times \pi} \approx 2,037$ revolutions.

5.46.  **Solution:** One rotation every 24 hours (or $24 \times 3600$ seconds) gives a linear speed $V$ equal to $V = 2 \times \pi \times R / T \approx 6.28 \times 6400 \times 1000 / (24 \times 3600) \approx 465$ m/sec.

**Answer:** $\approx 465$ m/sec.

5.47.  **Answer:** $\approx 108,000$ km/h.

5.48.  **Solution:** The length of a circle of radius $R$ is $2\pi R$, and the length of a bicycle wheel is $2\pi r$. The number of a wheel turns when the cyclist completes one loop is equal to $n = \frac{2\pi R}{2\pi r} = \frac{R}{r}$.

**Answer:** $R / r$.

5.49.  **Solution:** One meter a diameter corresponds to the circumference $\approx 6.3$ m. Therefore, when the wheel is pushed half a kilometer, it makes $n = \frac{500}{6.3} \approx 79$ revolutions.

**Answer:** 79 revolutions.

5.50.  **Solution:** $72$ km/h $= 1,200$ meters/minute. One revolution per minute corresponds to a distance equal to $2 \times \pi \times R$. Therefore, the number of revolutions per minute is equal to $V / (2 \times \pi \times R) \approx 159$ revolutions/minute.

**Answer:** 159 revolutions/minute.

5.51.  **Answer:** $10 / 13$. 

216
5.52. **Answer:** 34 (hint: $25 - 8 = 17$ is half of the total number of cars).

5.53. **Solution:** According to the problem, the wheel spins one revolution ($360^\circ$) per 20 seconds. Shawn starts to move from point A (in the figure below) under the condition that he takes 5 seconds to reach the top. Therefore, he gets to point B in 45 seconds, and the distance between point B and the ground is equal to 32 meters.

![Diagram of a wheel with points A and B](image)

**Answer:** 32 m.

5.54. **Solution:** A tip of a second hand makes one revolution per sixty seconds. One revolution corresponds with a circumference equal to $2 \times \pi \times L = 0.628$ m. So the speed of a second hand tip is $0.628 \div 60 \approx 0.01$ m/s or 1 cm/sec.

**Answer:** 1 cm/sec.

5.55. **Answer:** 0.1 degree/sec.

5.56. **Solution:** One 360-degree revolution of the minute hand corresponds with a circumference length equal to $2 \times \pi \times R$, where $R = \text{length of the minute hand}$. So $R = 1.25 \times 60 / (2 \times \pi) \approx 12$ centimeters.
Answer: 12 cm.

5.57. Solution: The circumference of the shaft is \( \pi D \approx 3 \times 0.2 = 0.6 \) meters. 20 revolutions correspond to the length \( 0.6 \times 20 = 12 \) meters. The depth of the well is 12 meters.
Answer: 12 m.

5.58. Solution: The time it takes the first cyclist to travel from point A to point B along a straight line is \( T_1 = \frac{AB}{V_1} \), and the time spent by the second cyclist, moving from point A to point B along the semicircle, is equal to \( T_2 = \frac{\pi \times AB}{2V_2} \). Because \( T_1 = T_2 \), the solution is \( \frac{V_2}{V_1} = \frac{\pi}{2} \).
Answer: \( \frac{V_2}{V_1} = \frac{\pi}{2} \).

5.59. Solution: When the little wheel makes twelve revolutions, the large gear makes \( \frac{12}{(32/8)} = 3 \) turns.
Answer: 3 turns.
5.60. **Solution:** Diameters of the two gears are related as 3:8. Hence, the ratio of their circumferences is also is 3 / 8. One turn of the small gear corresponds to 360°. Therefore, big gear turns by an angle 3 / 8 of 360° or 135°.

![Gears Diagram](image)

**Answer:** 135°.

5.61. **Answer:** Gear #1 turns at an angle ≈ 105°, gear #2 turns at an angle ≈ 171°, gear #3 turns at an angle of 249°, and gear #5 turns at an angle of 646°.

![Gears Diagram](image)

5.62. **Solution:** The distances the sled travels in seconds one, two, three, and so on are the terms of the arithmetic sequence. The first term of the sequence is equal to 2.5, the common difference of the arithmetic sequence is 5, and the sum of finite terms is 1,000. Let x = the number of terms of an arithmetic progression. Then (2, 5 + 5(x –
1) = the distance passed at the last second (the last term of the progression) and \[ \frac{2.5 + 2.5 + 5(x-1)}{2} \cdot x = \text{the total hill length (the progression sum).} \] This expression is equal to 1,000. The solution of the equation is \( x = 20 \).

**Answer:** 20 sec

5.63. **Solution:** The total distance the penny falls in 6 seconds is equal to 30 meters.

(The sum of six members of a finite arithmetic progression called an arithmetic series is
\[ S_6 = \frac{2a_1 + d(n-1)}{2} = \frac{2 \times 5 + 10 \times (6 - 1)}{2} = 30. \]

**Answer:** 30 m.

5.64. **Solution:** The time \( T \) to cross the x-axis is equal to \( T = \frac{BC}{V} \), where \( BC = \text{length of the right triangle ABC} \) and \( V = \text{speed of the ball at the braking string moment} \). \( BC = AB = L \). So, time \( T = \frac{L}{V} = 10 \text{ seconds} \).

**Answer:** 10 sec.

5.65. **Solution:** To estimate \( t \), it necessary to solve quadratic equation \( 0.1t^2 + 2t = 30 \). The positive root value of this equation is \( t = 10 \). So, the coil makes thirty turns per 10 seconds.
Answer: 10 sec.

5.66. Answer: 8 sec.

5.67. Solution: To find the solution, we have to solve the quadratic equation \(3 = 1.6 + 8t - 5t^2\). Roots of this equation are \(t_1 = 0.2\) sec and \(t_2 = 1.4\) sec. So, the ball will be at a height of more than three meters between \(t_1 = 0.2\) sec and \(t_2 = 1.4\) sec. \(\Delta = 1.4 - 0.2 = 1.2\) sec. 
Answer: 1.2 sec.

5.68. Solution: To find the solution, we have to solve the quadratic equation \(1.6 = 1.6 + 8T - 5T^2\). Roots of this equation are as following: \(T_1 = 0\) sec, \(T_2 = 1.6\) sec. So, the ball will be at a height of more than 1.6 m between \(0 \leq T \leq 1.6\) sec. 
Answer: \(0 \leq T \leq 1.6\) sec.

5.69. Solution: The equation for \(h\) shows that for \(T = 0\), \(h = 25\) m. It means that the balcony is located 25 m above the ground. Value \(h = 0\) corresponds with ground level. So, the value of \(T\) can be found with the equation \(h = 0 = 25 + 20T - 5T^2\). This quadratic equation has two roots, \(T_1 = 5\) and \(T_2 = -1\). A second solution as a negative does not make sense. Therefore, the correct solution is \(T = 5\) sec.
Answer: 5 sec.

5.70. Solution: The equation for \(h\) shows that for \(t = 0\), \(h = 25\) m. It means that the balcony is twenty-five meters above the ground. The value \(h = 0\) corresponds with ground level. So, the value of \(T\) can be found with the equation \(h = 0 = 25 - 20T - 5T^2\). This quadratic equation has one positive root \(T_1 = 1\). A second solution as a negative does not make sense. Therefore, the correct solution is \(T = 1\) second.
Answer: 5 sec.
Chapter 14
Math for Physics

Solutions and Answers

I. A law of conservation of mechanical energy in motion

6.1. Solution: At a height h above the ground, the ball has a potential energy equal to $E_p = mgh$. At the earth level, the potential energy is 0; however, the kinetic energy is equal to $E_k = \frac{mV^2}{2}$. According to the conservation of mechanical energy, $E_p + E_c = \text{const}$. Therefore, $mgh = \frac{mV^2}{2}$ or $V = \sqrt{2gh}$.

Answer: $V = \sqrt{2gh}$.

6.2. Solution: At the level h above the ground, the wheel has a potential energy equal to $E_p = mgh$. When the wheel rolls along the ground, the motion is determined by the combination of rotational and translational motions. The kinetic energy can be written as a sum of translational and rotational kinetic energy. The kinetic energy of the translational motion is $E_{k1} = \frac{mV_1^2}{2}$, where $V_1$ is the velocity of the transitional motion of the wheel center. The kinetic energy of the rotational motion is equal to $E_{k2} = \frac{mV_2^2}{2}$, where $V_2$ is the wheel
rotation speed. If there is no slipping, $V_1 = V_2$. At the ground level, $E_p = E_{k1} + E_{k2}$, or $V = \sqrt{gh}$.

Answer: $V = \sqrt{gh}$.

6.3. Answer: $V = \sqrt{gh}$.

6.4. Solution: The total initial energy of the particle is $E_{total} = E_p + E_k = \frac{4}{3}mgh$. At the level $h$ above the ground, the total initial energy is $E_{total1} = mgh$. At the level $h/4$, the total energy is $E_{total2} = E_{p2} + E_{k2} = \frac{mV^2}{2} + \frac{1}{4}mgh$. Because $E_{total1} = E_{total2}$, $V = \frac{1}{2}\sqrt{6gh}$.

Answer: $V = \frac{1}{2}\sqrt{6gh}$.

6.5. Solution: The mechanical energy is a constant value during the motion. Therefore, the equation that can evaluate the ball’s velocity is as follows: $mgh + \frac{mV_0^2}{2} = mgH$. This quadratic equation has the root $V_0 \approx 6.3$ m/sec.

Answer: $V_0 \approx 6.3$ m/sec.

6.6. Solution: The total energy $E_{total}$ of the ball at the height $H$ above the ground is equal to sum of the potential and kinetic energy $E_{total} = E_{p1} + E_{k1} = mgh + \frac{mV_x^2}{2}$. Because the initial kinetic energy
is \( E_{k1} = \frac{mV_x^2}{2} = \frac{1}{3} mgh \), the initial horizon velocity is \( V_x = \sqrt{\frac{2}{3} gh} \), and the total energy is \( E_{total} = \frac{4}{3} mgh \). When the cannonball strikes the earth, all potential energy is converted into kinetic energy, where

\[
E_{total} = \frac{4}{3} mgh = E_{k2} = \frac{mV^2}{2} \quad \text{and} \quad V = 2 \sqrt{\frac{2}{3} gh} \quad \text{or} \quad \frac{V_x}{V} = \frac{1}{2}
\]

The velocity of the ball at ground level is separated into two perpendicular components, as shown in the attached figure. The horizontal velocity component \((V_{x1})\) describes the influence of the velocity in displacing the projectile horizontally. The vertical velocity component \((V_y)\) describes the influence of the velocity in displacing the projectile vertically. The direction of the velocity \(V_{x1}\) is parallel to the earth at the height of \(h\) and at the ground level.

The angle between the direction of the velocity \(V_{x1}\) and the direction of the velocity \(V\) can be evaluated using the following method. The horizontal component \(V_{x1}\) does not change during the movement, because the gravity force direction is vertical. Thus, \(V_x = V_{x1}\), or \(\frac{V_{x1}}{V} = \frac{1}{2}\); therefore, the angle between the horizon and velocity direction when the ball strikes the earth is \(30^0\).

Answer: \(30^0\).
6.7. **Solution:** Because the ball is propelled from the ground at an angle of $45^0$ to the horizon, the vertical velocity component is equal to the horizontal component and equal to $V_y = \frac{20}{\sqrt{2}}$ m/sec. The total initial energy of the ball is

$$E_{\text{total}} = E_p + E_k = mgh + \frac{mV_x^2}{2} + \frac{mV_y^2}{2},$$

where $m$ is ball mass. At the maximum height, $E_{\text{total}} = E_p + E_k = mg(h + H) + \frac{mV_x^2}{2}$.

Therefore, $H = \frac{V_y^2}{4g}$ or $H = 10$ m.

**Answer:** 41.6 m.

6.8. **Solution:** Using the results of the previous problem, it can be seen that the time to reach the maximum height is $t = \frac{V_0}{g\sqrt{2}}$, and the total time of motion is $t_\Sigma = 2t = \frac{2V_0}{g\sqrt{2}}$. Therefore,

$$D = V_x \times t_\Sigma = \frac{V_0}{\sqrt{2}} \times \frac{2V_0}{g\sqrt{2}} = \frac{V_0^2}{g} = 40$$ m.

**Answer:** 40 m.

6.9. **Solution:** Because we have elastic collision, it can be written at the point A

$$m_1V_1^2 = m_2V_2^2 + m_1U^2 \quad (1),$$

 according to the problem,

$$m_1V_1 = m_2V_2 + m_1U \quad (2).$$

Substituting $U$ from expression (2) to (1),

$$V_2 = \frac{2m_1V_1}{m_1 + m_2}$$

can be obtained.
On the other hand, the speed value \( V_1 \) can be estimated from the law of conservation of energy \( V_1 = \sqrt{2gl} \). Thus, \( V_2 = \frac{2m_1 \times \sqrt{2gl}}{m_1 + m_2} \).

Based on this solution, we can estimate how high the second ball will rise above its starting point, namely \( m_2gh = \frac{m_2V_2^2}{2} \) or

\[
h = \frac{V_2^2}{2g} = \frac{4m_1^2 \times l}{(m_1 + m_2)^2} = 12.5 \text{ cm.}
\]

Answer: 12.5 cm.

**II. Liquids and Archimedes’ principle**

6.10. Solution: According to Archimedes’ principle the weight of the floating in the liquid object is equal to the buoyancy force, which is equal to weight of the object. In our case, total weight is determined by the weight of the raft and the maximum weight of an object that the raft can hold. Denote the weight of the raft as \( W_1 \), object weight as \( W_2 \), and buoyancy force as \( W_3 \). Thus, \( W_1 + W_2 = W_3 \), or \( W_2 = W_3 - W_1 \), or \( W_2/W_1 = W_3/W_1 - 1 \). The raft weight is \( W_1 = \rho_d \times V_d \times g \), where \( \rho_d \) =draft density. If the raft is completely immersed in the water and floats, the buoyancy force is equal to \( W_3 = \rho_w \times V_d \times g \), where
\[ \rho_w = \text{water density}. \] Therefore, \[ \frac{W_3}{W_1} = \frac{\rho_w}{\rho_d} = \frac{1}{0.8} \] and the ratio that we have to find is equal to \[ \frac{W_2}{W_1} = \frac{W_3}{W_1} - 1 = \frac{\rho_w}{\rho_d} - 1 = \frac{1}{0.8} - 1 = \frac{1}{4}. \] Thus, the raft can hold an object with a weight equal to 0.25 of the raft’s weight.

**Answer:** 0.25 of the raft’s weight.

**6.11. Solution:** Denote the weight value of the raft by \(W_1\), load weight by \(W_2\) and buoyancy force by \(W_3\). If the raft is floating, then \(W_1 + W_2 \leq W_3\). According to the problem condition, \(W_2 = W_1/2\). Therefore, if the raft is floating, then \[ \frac{W_2}{W_1} \geq \frac{3}{2}. \] On the other hand, it is known that \[ W_1 = \rho_d \times V_d \times g \] and \[ W_3 = \rho_w \times V_d \times g. \] The ratio \(W_3/W_1\) calculated from the relationship between the wood density and water density is equal to 2, which is greater than 3/2; therefore, raft will float under the problem conditions.

**Answer:** Yes.

**6.12. Solution:** The volume of the cylinder with height of \(H\) and radius of \(R\) is equal to \(\pi \times R^2 \times H\). In accordance with the law of Archimedes for floating cylinders:
\[ M_1 \times g = \rho \times g \times \pi \times R_1^2 \times H_1; \]
\[ M_2 \times g = \rho \times g \times \pi \times R_2^2 \times H_2. \]

Where \( \rho = \) the density of the water. Therefore,

\[ \frac{H_1}{H_2} = \frac{M_1 \times R_2^2}{M_2 \times R_1^2}. \]

**Answer:** \( \frac{H_1}{H_2} = \frac{M_1 \times R_2^2}{M_2 \times R_1^2}. \)

6.13. **Solution:** The buoyant force is equal to

\[ B = \frac{4}{3} \times \pi \times R_E^3 \times \rho_L \times g . \]

The ball weight is

\[ W = \frac{4}{3} \times \pi \times (R_E^3 - R_i^3) \times \rho_M \times g . \]

For a floating ball, \( W = B. \)

Therefore,

\[ \frac{R_i}{R_E} = \sqrt[3]{\frac{\rho_M - \rho_L}{\rho_M}}. \]

**Answer:** \( \frac{R_i}{R_E} = \sqrt[3]{\frac{\rho_M - \rho_L}{\rho_M}}. \)

6.14. **Solution:** Let’s say that \( V_b \) is the volume of the human body, \( \rho_b \)

is the density of the human body, and \( \rho_s \) is the density of seawater.

According to Archimedes’ principle, any object immersed in fluid is

buoyed up by a force equal to the weight of the fluid displaced by the

object. Therefore, the weight of the human body immersed in

seawater is \( W_s = \rho_b \times V_b \times g - \rho_s \times V_b \times g . \)

The weight of the human body in air is equal to \( W_a = \rho_b \times V_b \times g . \)

Their ratio is

\[ \frac{W_s}{W_a} = \frac{\rho_b - \rho_s}{\rho_b} = 1 - \frac{\rho_s}{\rho_b} = 1 - \frac{W_s}{W_a}. \]

According to the

problem, \( \frac{W_s}{W_a} = \frac{1}{30} . \)

Thus, \( \frac{\rho_s}{\rho_b} = \frac{29}{30} \approx 0.967 . \)
Answer: 0.967.

6.15. Answer: The ratio is about 30.

6.16. Solution: Both geometry models float in the liquid, and therefore, the weight of the sphere $F_G$, as well as the weight of the cone, has to be equal to the buoyant force $F_b$. Because the weights of these models are the same, the buoyant forces must also be the same. Buoyant force is proportional to the model volume, and thus, the cone volume $V_1 = \frac{\pi R_1^3}{3}$ should be equal to the sphere volume $V_2 = \frac{4}{3} \times \pi R_2^3$, or $\frac{\pi R_1^3}{3} = \frac{4}{3} \times \pi R_2^3$, which can also be stated as

$$\frac{R_1}{R_2} = \frac{3}{\sqrt[3]{4}}.$$ 

Answer: $\frac{R_1}{R_2} = \frac{3}{\sqrt[3]{4}}$.

6.17. Solution: Initially, the volume of water in the first cylinder is $\pi R^2H$. When the cylinders are connected with the pipes, the total volume in all four cylinders is equal to $\pi R^2 h \times (1 + 1/4 + 1/16 + 1/64) = \pi R^2 h \times 85/64$. This value has to be the same as that of the original volume. Thus, $h = H \times 64/85 \approx 0.75H$. 

Answer: $\frac{R_1}{R_2} = \frac{3}{\sqrt[3]{4}}$. 

230
Figure 1

Answer: $h/H = 64/85 \approx 0.75$.

Figure 2

Answer: $h/H = 64/85 \approx 0.75$.

6.18. Solution: Let $x$, $y$, and $z$ denote the numbers that we have to evaluate. They are consecutive numbers of the geometric sequence. Use the following property of this sequence: $y^2 = x \times z$. Since the numbers $x$, $y$, and $(z-4)$ are consecutive terms of an arithmetic progression, $y = ((x + (z-4))/2$. Finally, the numbers $x$, $(y-1)$, and $(z-5)$ are consecutive terms of the geometric sequence; thus, $(y-1)^2 = x(z-5)$. Therefore, we obtain the following system of equations:

\[
\begin{cases}
y^2 = xz \\
x + z - 4 = 2y \\
(y-1)^2 = x(z-5)
\end{cases}
\]

This system has only one solution in integer numbers: $x=1$, $y=3$, $z=9$. Answer: 1, 3, 9.
III. Electrical circuits, series, and parallel connections (Ohm’s law)

6.19. **Answer:** 10R.

6.20. **Solution:** According to the problem, resistor values represent a geometric progression with a common ratio of $1/2$. The total resistance of series-connected resistors is equal to the sum of all resistor values. Therefore,

$$R_\Sigma = R + R/2 + R/4 + \ldots + R/512 =$$

$$R \times \left( 1 - \left( \frac{1}{2} \right)^{10} \right)$$

$$R_\Sigma = \frac{R \times \left( 1 - \left( \frac{1}{2} \right)^{10} \right)}{1 - \frac{1}{2}} = 2R \times \left( 1 - \left( \frac{1}{2} \right)^{10} \right) \approx 2R.$$

**Answer:** 2R.

6.21. **Answer:** R/10.

6.22. **Solution:** If we have two resistors, $R$ and $R/2$, connected in parallel, then the total resistance is $R/3$.

If we have three resistors—$R$, $R/2$, and $R/4$—connected in parallel, then the total resistance is $R/7$.

If we have four resistors—$R$, $R/2$, $R/4$, and $R/8$—connected in parallel, then the total resistance is $R/15$. Therefore, for 10 resistors connected in parallel, the total resistance is $R_{\Sigma(10)} = \frac{R}{2^{10} - 1}$.

**Answer:** $R_{\Sigma(10)} = \frac{R}{2^{10} - 1}$.

6.23. **Answer:** Based on the results of the previous problem, $R_{\Sigma(100)} = \frac{1}{2^{100} - 1}$.
6.24. **Solution:** If we have two resistors, $R$ and $R/3$, connected in parallel, then the total resistance is $R/4$.

If we have three resistors connected in parallel, $R$, $R/3$, and $R/5$, then the total resistance is $R/9$.

If we have four resistors connected in parallel, $R$, $R/3$, $R/5$, and $R/7$, then the total resistance is $R_{\Sigma(4)} = \frac{R}{2^4} = \frac{R}{16}$. Therefore, for 100 resistors connected in parallel the total resistance is $R_{\Sigma(100)} = \frac{R}{100^2}$.

**Answer:** $R_{\Sigma(100)} = \frac{R}{100^2}$.

6.25. **Solution:** Denote the resistance of the unknown resistor as $x$. In this case, the total resistance of two parallel resistors is $\frac{R \times x}{2} = \frac{R}{3}$.

The solution of this linear equation is $x = R$.

**Answer:** $R$.

6.26. **Solution:** All possible options are shown in the figure below.

[Diagram of circuit configurations]
Connection #1 gives equivalent resistance of $R_\Sigma=3R$; connection #2 gives equivalent resistance of $R_\Sigma=R/3$; connection #3 gives equivalent resistance of $R_\Sigma=2R/3$; and connection #1 gives equivalent resistance of $R_\Sigma=3R/2$.

6.27. **Solution**: According to the equivalence between circuits in Figure 2, the circuit in Figure 1 can be presented in Figure 3.

![Figure 1](image1.png)

![Figure 2](image2.png)

$$R_1 = \frac{R_{12} \times R_{31}}{R_{12} + R_{31} + R_{23}}$$

$$R_2 = \frac{R_{12} \times R_{23}}{R_{12} + R_{31} + R_{23}}$$

$$R_3 = \frac{R_{23} \times R_{31}}{R_{12} + R_{31} + R_{23}}.$$
Figure 3

The total resistance of the circuit in Figure 3 between points A and B is $R_{\Sigma} = 3R/2$.

**Answer:** $R_{\Sigma} = 3R/2$.

6.28. **Solution:** If the value of one resistor is reduced by a factor of 2, then the total resistance of two parallel resistors becomes $R/3$. Therefore, we have to add $R/6$ in series with a parallel connection to obtain $R/2$.

**Answer:** $R/2$.

6.29. **Solution:** The resistance of $(N-1)$ identical parallel-connected resistors is $R/(N-1)$, where $R$ is the resistance of one resistor. The next step is to connect an additional resistor with a value of $2R$ to this circuit in parallel. Now, the total resistance is $2R/(2N-1)$.

**Answer:** $2R/(2N-1)$.

6.30. **Solution:** The resistance of $(N-2)$ identical resistors connected in parallel is $R/(N-2)$. Add another resistor (in parallel) with a resistance of $2R$. The result is $2R/(2N-3)$. Now, add another one with unknown value $x$. The result is as follows:

$$R_{\Sigma} = \frac{2R}{(2N-3)} \times x$$

According to the problem condition, $R_{\Sigma} = \frac{R}{N}$, i.e.
\[ R_x = \left(\frac{2R}{2N-3}\right) \times x = R \frac{2R}{(2N-3)+x} = \frac{R}{N}. \]

This is an equation with one unknown. The solution for this equation is \( x = \frac{2R}{3} \).

**Answer:** \( \frac{2R}{3} \).

6.31. **Solution:** One possible solution: Suppose we have \( N \) parallel-connected identical resistors. This circuit is connected with \( Q \) series-connected identical resistors. The total resistance of this circuit is

\[ \frac{R}{N} + QR = R \left(\frac{1+QN}{N}\right). \]

Let us assume that \( N=11 \) and \( 1+N\times Q = 144 \). Then, \( Q=13 \). So, the solution to the problem is as follows: The first circuit contains 11 parallel-connected resistors, the second circuit contains 13 series-connected resistors, and finally these two circuits are connected in series.

6.32. **Answer:** \( \frac{4C}{3} \).

6.33. **Answer:** \( \frac{46C}{7} \).

6.34. **Answer:** \( \frac{3C}{7} \).

6.35. **Answer:**

![Diagram of circuits](image)
Connection #1 gives an answer of $C_\Sigma = C/3$, connection #2 gives $C_\Sigma = 3C$, for connection #3, the total capacitance is $C_\Sigma = 3C/2$, and the equivalent capacitor for circuit #4 is $C_\Sigma = 2C/3$.

IV. The mathematics of lenses

6.36. Solution: Let the image distance be $d_i = x$. According to the problem, $d_o = 50$ cm and $FO = 8$ cm; therefore, $\frac{1}{50} + \frac{1}{x} = \frac{1}{8}$. The root of this linear equation is $x = 12.5$ cm.

Answer: $d_i = 12.5$ cm.

6.37. Solution: If $M = 10$ and the distance between the lens and the image is 600 cm, then the distance $d_o$ from the object to the lens is $600$ cm/10 = 60 cm. Now apply the lens equation $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ to solve the problem. Here, $\frac{1}{600} + \frac{1}{60} = \frac{1}{f}$. Thus, $f \approx 54.5$ cm.

Answer: $\approx 54.5$ cm.

6.38. Solution: Based on the previous solution, we have $d_i = 12.5$ cm. Triangles ABO and COD are similar; therefore, $AB/AO = DC/OC$. Thus, $DC = h_i = AB \times OC / AO = 5 \times 12.5 / 50 = 1.25$ cm.
Answer: 1.25 cm.

6.39. **Solution**: The value of the distance between the lens and the image based on the lens equation is $OC = 60$ cm. The height $h_i$ of the image is evaluated from the similarity of the triangles $ABO$ and $CDO$ is $h_i = DC = AB \times OC / AO = 10$ cm.

Answer: 10 cm.

6.40. **Solution**: The value of the distance between the lens and the image can be evaluated from the lens equation $\frac{1}{20} + \frac{1}{OC} = \frac{1}{45}$. Thus, $OC = -36$ cm. The sign (-) means that the image and original object appear on the same side from the lens. The height $h_i$ of the image is calculated from the similarity of the triangles $ABO$ and $CDO$ is $h_i = DC = AB \times OC / AO = 9$ cm.
6.41. **Solution:** Let the image distance be \( d_i = x \). According to the problem, \( d_o = 9 \) cm and \( FO=6 \) cm; therefore, \( \frac{1}{d_o} + \frac{1}{x} = -\frac{1}{6} \). The root of this linear equation is \( x = -3.6 \) cm. The sign (-) means that the image and original object appear on the same side of the lens.

Answer: \( d_i = 12.5 \) cm.

6.42. **Solution:** Initially, we ignore lens 2 and evaluate the location of the image from lens 1 (see the figure below). Apply the lens equation to the object that is in front of lens 1—

\[
\frac{1}{p_1} + \frac{1}{i_1} = \frac{1}{f_1}
\]

It can be found that the distance equal \( i_1 = -10 \) cm, which signifies that virtual image \( I_1 \) is on the same side of the lens as the object and has the same orientation. Now, treat image \( I_1 \) as object \( O_2 \) and apply the lens equation to evaluate the image that produces this object when using lens 2. Because the distance between object \( O_2 \) and lens 2 is \( p_2 = 10 \) cm...
cm+8 cm=18 cm, we can calculate the value of $i_2$ from the equation \[ \frac{1}{p_2} + \frac{1}{i_2} = \frac{1}{f_2}, \]
where $p_2=18$ cm and $f_2=9$ cm. This linear equation has a root $i_2=18$ cm. Thus, the real image produced by lens 2 is located on the distance 18 cm from lens 2.

\[ \text{Answer: 18 cm.} \]

6.43. **Solution:** Magnify the power according to the formula \[ M = 1 + \frac{25}{f} \]
is 6.
The near point distance $D$ is the nearest to the eye distance at which an object is clearly focused on the retina when the accommodation of the eye is at the maximum. The magnification power for the image at the near point is $M = 1 + \frac{25}{f}$, where $f$ is the magnifier’s focal distance. The figures above show the object and eye with and without a magnifier.

Answer: 6.

6.44. Solution: When the object is placed 25 cm from the lens, we want the image to be 100 cm away on the same side of the lens (so the eye can focus on it), and so the image is virtual. To estimate the lens power, we use a thin converging lens equation (problem #47) with $d_0=25$ cm and $d_i=-100$ cm. The negative sign means that the image is located on the same side of the lens as the original object (problem #51).

Given that $d_0=25$ cm and $d_i=-100$ cm, the thin lens equation gives $f=33$ cm=0.33 m. The power $P$ of the lens is $P=1/f=+3D$.

Answer: 3D.

6.45. Solution: According to the magnification formula for the compound microscope, $M\approx 129$ can be calculated.
The microscope consists of one lens, the objective, that has a very short focal length \( f_0 < 1 \text{ cm} \), and a second lens, the eyepiece, that has a focal length \( f_e \) of a few centimeters. The two lenses are separated by a distance \( L \) that is much greater than either \( f_0 \) or \( f_e \). The object, which is placed just outside the focal point of the objective, forms a real, inverted image at \( I_1 \), and this image is located at or close to the focal point of the eyepiece. The eyepiece, which serves as a simple magnifier, produces a virtual, inverted image of \( I_1 \) at \( I_2 \). The overall magnification of the compound microscope is defined as \( M \approx \frac{25L}{f_o f_e} \).

Answer: 129.

V. The law of reflection

6.46. Solution: According to the known law of reflection, the angle between the incident ray and the normal (the angle of incidence) is equal to the angle between the reflected ray and the normal (the angle of reflection). The law of reflection is demonstrated in the attached figure, where line OA is a normal to the horizontally placed mirror and line BC is a normal to the inclined mirror. Following from the figure, \( \epsilon = 2\beta + 2\gamma \) and \( \alpha + 90^\circ - \beta + 90^\circ - \gamma = 180^\circ \), or \( \alpha = \beta + \gamma \). Thus, \( \epsilon = 2\alpha \) and does not depend on the incident ray angle \( \beta \).
Answer: $\epsilon = 2\alpha$.

6.47. Solution: The angle $\angle \text{HOG} = \angle \text{BOD} = 30^\circ$. Therefore, angle $\angle \text{FOH} = \angle \text{HOE} = 40^\circ$. In addition, angle $\angle \text{FOG} = 70^\circ$. Therefore, angle COF is equal to $90^\circ - 70^\circ = 20^\circ$.

Answer: $20^\circ$.

6.48. Solution: According to the magnification formula for the compound microscope, $M \approx 129$ can be calculated.
The angles can be found as follows: Because the reflected wave falls vertically downwards, angle $\gamma$ between the reflected ray OA and horizon OX is equal to $90^0$. Moreover, the total angle is $90^0 + \alpha$, where $\alpha=60^0$. On the other hand, angle $\beta = \varepsilon$ (law of reflection), and therefore the sum of the angles $90^0 + \alpha + 2\beta = 180^0$, $\beta = (90^0 - \alpha)/2$ and $\alpha + \beta = \alpha + (90^0 - \alpha)/2 = 45^0 + \alpha/2 = 75^0$.

**Answer:** The mirror should be placed at an angle to the horizon equal to $75^0$.

6.49. **Answer:** 5 times. The solution is shown in the figure below.
Chapter 15
Probability and Statistics

Solutions and Answers

7.1. Answer: 3/5.

7.2. Answer: 2/5.

7.3. Solution: The probability that the first apple will be red is 3/5. After you select a red apple, there will only be four apples left in the basket, two red and two yellow. The probability that you select a red apple in this case is 1/2. Thus, the probability that the first and second apples are red is equal to \( \frac{3}{5} \times \frac{1}{2} = \frac{3}{10} \).
Answer: 0.3.

7.4. Solution: The probability that the first apple will be yellow is 2/5. After you select a yellow apple, there are only four apples left in the basket, three red and one yellow. The probability that you will select a yellow apple in this case is 1/4. Thus, the probability that you select both yellow apples is equal to \( \frac{2}{5} \times \frac{1}{4} = \frac{2}{20} = \frac{1}{10} \).
Answer: 0.1.

7.5. Solution: The probability that the first apple will be red is 3/5. After you select a red apple, there are only four apples left in the basket, two red and two yellow. The probability that you will select a yellow apple in this case is 1/2. Thus, the probability that the first apple is red and second is yellow is \( \frac{3}{5} \times \frac{1}{2} = \frac{3}{10} \).
Answer: 0.3.

7.6. Solution: The probability that the first apple will be yellow is 2/5. After you pick a yellow apple, there will only be four apples left in the basket, three red and one yellow. The probability that you
select a red apple in this case is 3/4. Thus, the probability that the first apple is yellow and the second is red is \((2/5) \times (3/4) = 3/10\).

Answer: 0.3.

7.7. **Solution:** According to the problem, the first apple selected from the basket has to be yellow. The probability of such an event is 2/5. Then there are four apples in the basket—three red and one yellow. The probability that the second apple taken out of the basket will be red is 3/4. If the second apple taken out of the basket is red, then there are three apples left in the basket, two red and one yellow. The probability that the third apple selected from the basket will be red is 2/3. So, the total probability that second and third apples selected from the basket will be red is \((2/5)\times(3/4)\times(2/3)=1/5\).

Answer: 0.2.

7.8. **Solution:** Event #1: Victor selects a good apple and then a spoiled apple. Event #2: Victor selects a spoiled apple and then good apple. The probability of event #1 is \(P_1= (4/5)\times(1/4)\). The probability of event #2 is 1/5. The total probability is \(P_\Sigma=P_1+P_2=2/5\).

Answer: 2/5.

7.9. **Solution:** The probability of picking a red apple from the first basket is 1/6 and that of selecting a yellow apple 5/6. The probability of selecting a red apple from the second basket is 8/12 and that of choosing a yellow apple 4/12. Thus, the total probability of selecting one red and one yellow apple is \((1/6)\times(4/12)+(5/6)\times(8/12)=11/18\).

Answer: 11/18.

7.10. Answer: 0.85.

7.11. **Solution:** The required probability is equal to \((3/8) \times (5/7) + (5/8) \times (3/7) = 30/56=15/28\).

Answer: 15/28.

7.12. **Solution:** Event #1: The first selected ball is white; event #2: the second selected ball is black. The probability of event #1 is 5/14. If we pick a white ball, then there will be 13 left in the bag and the probability of event #2 is 9/13. The total probability of events #1 and #2 is \((5/14)\times(9/13)\approx0.247\).
7.13. **Solution:** The required probability \( P \) is equal to the following:
\[
P = \frac{1}{3} \times \frac{2}{3} + \frac{1}{3} \times \frac{3}{4} + \frac{1}{3} \times \frac{1}{2} = \frac{23}{26}.
\]
**Answer:** \( 23/26 \).

7.14. **Answer:** \( P = (1/6)^3+(1/6)^4+(1/6)^5+(1/6)^6 \approx 0.006 \).

7.15. **Solution:** Numbered tickets: 1, 2, 3, 4 ... 9, 10. Tickets whose number is a multiple of 3 are 3, 6, and 9, while those with a multiple of 5 are 5 and 10. Therefore, the probability that the ticket drawn has a number that is a multiple of 3 or 5 is equal to 6/10.

**Answer:** 0.6.

7.16. **Solution:** There are 11 total integers between 30 and 40. The total number of cards with an integer multiple of three is equal to four (30, 33, 36, 39). Thus, the required probability is 4/11.

**Answer:** 4/11.

7.17. **Answer:** 1/6.

7.18. **Answer:** 1/24.

7.19. **Solution:** If the number is divisible by 3 and 2, then it is divisible by 6. The total number of integers divisible by 6 in the interval from 1 to 100 is equal to 16. Thus, the required probability is 16/100.

**Answer:** 0.16.

7.20. **Answer:** 1-(1/2)^3=7/8.

7.21. **Solution:** According to the problem, there are four possible outcomes, as follows: GGG, BBB, BBG, and BGG. Out of these four outcomes, three are favorable. Thus, the probability is 3/4.

**Answer:** 0.75.

7.22. **Solution 1:** There are the following event options: E1: \{B B B\}, E2: \{B G G\}, E3: \{G B G\}, E4: \{G G B\}, E5: \{B B G\}, E6: \{B G
B}, and E7: \{G\ B\ B\}. In case event E1, the family has three boys; in events E2, E3, and E4, the family has one boy; and in events E5, E6, and E7 the parents have two boys. The probability \( P_\Sigma \) that at least one of the kids is a boy is equal to the sum of all possible probabilities, namely the probability \( P_3 \) that all three kids are boys plus probability \( P_2 \) that two of the kids are boys plus probability \( P_1 \) that one of the three kids is a boy.

\[
P_\Sigma = P_3 + P_2 + P_1 = 0.51^3 + 3 \times 0.51^2 \times 0.49 + 3 \times 0.49^2 \times 0.51 = 0.88
\]

Solution 2: The probability that parents have at least one boy is equal to 1 minus the probability that all of them are girls, or \( 1 - 0.49^3 = 0.88 \).

Answer: 0.88.

7.23. Answer: 0.13.

7.24. Answer: \( 0.49^3 \approx 0.11 \).

7.25. Answer:

\[
P = 0.51 \times 0.51 \times 0.49 + 0.51 \times 0.49 \times 0.51 + 0.49 \times 0.51 \times 0.51 = 3 \times 0.49 \times 0.51^2 \approx 0.38.
\]

7.26. Answer: \( P = 0.51 \times 0.51 \times 0.49 + 0.51 \times 0.51 \times 0.51 \approx 0.26 \).

7.27. Solution: The following options are available:

a. \( P_1, V_2, A_3 \);

b. \( P_1, V_3, A_2 \);

c. \( V_1, P_2, A_3 \);

d. \( V_1, P_3, A_2 \);

e. \( A_1, P_2, V_3 \); and

f. \( A_1, P_3, V_2 \).

In total, there are 6 different options. As can be observed, the required probability is \( 2/6 = 1/3 \).

Answer: \( 1/3 \).

7.28. Solution: The probability that the biathlete will hit the target four times is \( \left( \frac{4}{5} \right)^4 \). The probability that he will not hit the target at all is \( 1/5 \). Thus, the probability that the biathlete will hit the target only
three times is \( \left( \frac{4}{5} \right)^3 \times \frac{1}{5} \). Therefore, the probability that he will hit
the target at least three times is equal to \( \left( \frac{4}{5} \right)^4 + 4 \times \left( \frac{4}{5} \right)^3 \times \frac{1}{5} \approx 0.8 \).

Answer: 0.8.

7.29. Solution:
a. The probability that only the first hunter killed the bear is \( 0.3 \times 0.6 = 0.18 \);
b. The probability that only the second hunter killed the bear is \( 0.7 \times 0.4 = 0.28 \);
c. The probability that a bear was killed by the first and second
hunters is \( 0.3 \times 0.4 = 0.12 \);
d. The probability that the bear was not kill by the first and the
second hunter is \( 0.7 \times 0.6 = 0.42 \).

Answer: \( P_1=0.18, P_2=0.28, P_3=0.12, P_4=0.42 \).

7.30. Solution: The probability that the bear was not killed by the first
hunter is 0.7; the probability that it was not killed by the second
hunter is 0.6; and the probability that the third hunter killed it
regardless of the first two is 0.5. Thus, the probability that the bear
was killed only the third hunter is \( 0.7 \times 0.6 \times 0.5 = 0.21 \).

Answer: 0.21.

7.31. Solution:

\[
P_2 = 0.1 \times 0.2 \times 0.3 + 0.1 \times 0.2 \times 0.7 + 0.1 \times 0.8 \times 0.3 + 0.9 \times 0.2 \times 0.3 + 0.9 \times 0.8 \times 0.3 + 0.9 \times 0.2 \times 0.7 + 0.1 \times 0.8 \times 0.7 = 0.496.
\]

Answer: 0.496.

7.32. Solution: The probability that a vehicle travelling along the
highway is a truck is 0.6, while the probability that it is a passenger
car is equal to 0.4. The probability that truck or car stops for gas is
\( 0.6 \times 0.1 + 0.4 \times 0.2 = 0.14 \).

Answer: 0.14.

7.33. Solution: The probability \( P=0.6 \times 0.9 + 0.4 \times 0.8 = 0.86 \).

Answer: 0.86.
7.34. **Solution:** Let us assume that the number in the empty cell is X. Then, the average time watching TV is 
\( \frac{0 \times 1 + 1 \times 9 + 2 \times X + 3 \times 4 + 4 \times 1}{25} = 1.8 \). Solving of this equation gives 
\( X = 10 \).

Answer: 10 hours.

7.35. **Solution:** Let the number of tests be N and the total score received for the N-1 test set be A. Then, 
\( \frac{(A+95)}{N}=90 \quad (1) \), \( \frac{(A+70)}{N}=85 \quad (2) \). The solution to this pair of equations is 
\( N = 5 \).

Answer: 5.

7.36. **Solution:** The total score achieved by the student is \( A_1 = 42 \) (the average score is 3.5). The total score for an average of 4 is \( A_2 = 48 \). Thus, the difference of \( A_2 - A_1 = 6 \). Therefore, to increase the average from 3.5 to 4, the student has to increase his grade in six subjects.

Answer: 6.

7.37. **Solution:** Let us divide the data from the table into three groups, with four numbers in each group. Let us calculate the average in each group. The first average is \( \frac{20 + 25 + 15 + 10}{4} = 15 \), the second average is \( \frac{25 + 30 + 25 + 20}{4} = 25 \), and the third average is \( \frac{30 + 35 + 45 + 30}{4} = 35 \). Comparing these averages, we can see a tendency of increasing noise with time.

7.38. **Solution:** The average of these data is \( X_{av} = 39.2 \). The mean absolute deviation is estimated as
\[
X_D = \frac{\sum_{i=1}^{N} |X_i - X_{av}|}{N} = 15.4.
\]

Answer: 15.4.


7.40. **Solution:** Let the numbers be \( X_1 = 24 \), \( X_2 = 8 \), and \( X_3 = \text{unknown} \). The formula for the absolute median deviation can be expressed as follows:
\[ X_D = \frac{1}{3} \left( |X_1 - X_{av}| + |X_2 - X_{av}| + |X_3 - X_{av}| \right), \]

where

\[ X_{av} = \frac{X_1 + X_2 + X_3}{3}. \]

Therefore,

\[ X_D = \frac{1}{3} \left( \left| 8 + \frac{24 + X}{3} \right| + \left| 24 - \frac{8 + 24 + X}{3} \right| + \left| X_3 - \frac{8 + 24 + X}{3} \right| \right) \]

or

\[ \left| 8 + X_3 \right| + \left| 40 - X_3 \right| + \left| 2X - 32 \right| = 54. \]

This is the equation with respect to \( X_3 \). To find the solution, we have to investigate the following intervals, with each of the following parameters for \( X_3 \):

a. \( X_3 > 40 \);

b. \( 16 < X_3 < 40 \);

c. \( -8 < X_3 < 16 \); and

d. \( X_3 < -8 \).

According to the problem, options “a” and “b” and “d” do not satisfy to the condition of the problem; therefore option “c” should be evaluated.

Open the brackets in the equation for option “c.” Then, \( 8 + X_3 + 40 - 2X_3 = 54 \) or \( X_3 = 13 \).

Answer: \( X_3 = 13 \).

7.41. Solution: Let us average data for random \( X_1, X_2, X_3, \) and \( X_4 \), as well as for \( Y_1, Y_2, Y_3, \) and \( Y_4 \). Then, \( X_{av1} = 2, Y_{av1} = 2; X_{av2} = 5, Y_{av2} = 8; X_{av3} = 13, Y_{av3} = 8; \) and \( X_{av4} = 10, Y_{av4} = 2 \). Thus, the quadrilateral ABCD has the following coordinates: A (2,2); B(5,8); C(13,8); D(10,2); E(5,2). The drawn quadrilateral is a parallelogram with the height equal to \( BE = 6 \) and lengths of \( AB = BC = 8 \). Therefore, the area of the parallelogram is equal to 48.
Answer: 48 cm.

7.42. Solution: After averaging the data in the tables, the following values can be obtained: $X_{1av}=2$, $Y_{1av}=2$; $X_{2av}=5$, $Y_{2av}=8$; and $X_{3av}=10$, $Y_{3av}=2$. Thus, the triangle ABC has the following coordinates: A (2,2); B(5,8); C(10,2); D(5,2). The drawn form is a triangle with a height of BE=6. Therefore, the area of the triangle is 24 cm.

Answer: 24 cm.

7.43. Solution: Ohm’s law gives the relationship between the voltage in the circuit, current, and resistance of $V = I \times R$. We know that the value of $V=10$ volts. The data for the current are in the table. The table below shows the values for the resistance, calculated from the measured current.

<table>
<thead>
<tr>
<th>Current I</th>
<th>1.2</th>
<th>0.8</th>
<th>1.4</th>
<th>0.9</th>
<th>1.1</th>
<th>0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistance $R\approx$</td>
<td>8.3</td>
<td>12.5</td>
<td>7.1</td>
<td>11.1</td>
<td>9.1</td>
<td>16.7</td>
</tr>
</tbody>
</table>

The average value of the resistance based on the measurement result is $R\approx10.8$ ohm.

Answer: $R\approx10.8$ ohm.

7.44. Solution: If the volume of the cubic container doubles, the linear dimensions of the cube will increased by a factor of $\sqrt[3]{2}$. Thus, the average distance between gas molecules becomes $\sqrt[3]{2} \times L$.

Answer: $\sqrt[3]{2} \times L$.

7.45. Answer: 1/3.
7.46. **Solution:** The probability \( P \) (Dart lands in the shaded region) = \( \frac{\text{Shaded area}}{\text{Area of the dart board}} = \frac{\pi \times R^2}{(2R)^2} = \frac{\pi}{4} \approx 0.75 \).

Answer: \( \approx 0.75 \).

7.47. **Solution:** The probability \( P = \frac{\text{Area of square}}{\text{Area of the rectangle}} = \frac{100}{100 \times 125} = 1/125. \)

Answer: 0.008.

7.48. **Solution:** Let \( PQ = 2X \) and \( QR = 2Y \). Then, \( AQ = X \) and \( QB = Y \).

The area of rectangle \( PQRS = 2X \times 2Y = 4XY \). The area \( AQB = XY/2 \). The probability of striking a shaded region is \( \frac{2XY}{4(XY)} = 1/2 \).

Answer: 1/2.

7.49. **Solution:** The graph shown below demonstrates Shawn’s arrival time in relation to Alex’s arrival time. The shaded area is the area where the wait time is less than 3 minutes. The whole graph area is 25 units, while the shaded area is 9 units\(^2\). The probability of waiting less than 3 minutes is \( 3/25 \) or 12%.

Answer: 12%. 

253
Chapter 16

Work Problems

Solutions and Answers

8.1. Answer: 344 parts.

8.2. Solution: If the first door is open, the people leave the hall through this door with a speed equal to \( \frac{N}{20} \). If the second door is open, the rate is \( \frac{N}{30} \). When we open both doors, the hall will be empty in

\[
T = \frac{N}{\frac{N}{20} + \frac{N}{30}} = \frac{20 \times 30}{50} = 12.
\]

Answer: 12 min.

8.3. Answer: 3 hours and 44 minutes (solution is identical to that of problem 8.2).

8.4. Solution: According to the problem, the rate \( V \) of filling the pool is equal to

\[
V = \frac{C}{5} - \frac{C}{9},
\]

where \( C \)=pool capacity. Let us assume that the time of emptying the pool is \( x \). Then,

\[
\frac{C}{5} - \frac{C}{9} = \frac{C}{x} \quad \text{or} \quad x = \frac{5 \times 10}{10 - 5} = 10.
\]

Answer: 7 hours.

8.5. Solution: Let the total amount of work to be performed equal \( A \). Then, the speed rate of the first worker is \( \frac{A}{t_1} \), where \( t_1 \)=time to complete the work for the first worker. The rate of the second worker
is \( \frac{A}{t_2} \), where \( t_2 \) is the time to complete the work for the second worker. Working together, they will do the job in

\[
t = \frac{A}{\frac{A}{t_1} + \frac{A}{t_2}} = \frac{t_1 \times t_2}{t_1 + t_2}.
\]

Knowing the values of \( t \) and \( t_2 \), we obtain \( t_1 = 10 \) hours.

**Answer:** 10 hours.

**8.6. Solution:** Denote the time to complete the fence painting for Igor as \( t_1 \), for Peter, as \( t_2 \), and for Vladimir, as \( t_3 \). Denote the total amount of work that the friends need to do as \( A \). Then, according to the problem,

\[
\begin{align*}
\frac{A}{t_1} + \frac{A}{t_2} &= \frac{A}{9} \quad (1) \\
\frac{A}{t_2} + \frac{A}{t_3} &= \frac{A}{12} \quad (2) \\
\frac{A}{t_3} + \frac{A}{t_1} &= \frac{A}{18} \quad (3)
\end{align*}
\]

and the sum of three equations is

\[
\frac{1}{t_1} + \frac{1}{t_2} + \frac{1}{t_3} = \frac{1}{8} = \frac{1}{T},
\]

where \( T \) is the time to complete the work if all friends work together. Thus, \( T = 8 \).

**Answer:** 8 hours.

**8.7. Solution:** Let the order volume be \( V \); the time to complete the order for the first company is \( X \) and that for the second company is \( X + 4 \). According to the problem, working together, they can complete
work that is five times greater in 24 days. So, \( \frac{5V}{V} = 24 \). 
\[ \frac{5V}{V + \frac{V}{X + 4}} \]
This is a quadratic equation that has a positive root equal to 8.

Answer: Company (A) completes the order in 8 days and company (B) in 12 days.

8.8. **Solution:** The son completed his work (1 hectare) in \( 60/0.75 = 80 \) minutes. During this time, his father plowed \( \frac{1}{2 \times 60} \times 80 = 2/3 \) ha. Thus, the remainder of the field equals 1/3 ha that they plow together. The time they spend plowing this part of the field is 
\[ \frac{1}{3} + \frac{1}{2 + \frac{3}{4}} = \frac{4}{15} \text{, or 16 minutes.} \]
Thus, the total time to complete all of the work is 96 minutes.

Answer: 96 min.

8.9. **Solution:** Denote the aquarium volume as \( V \), the filling time if the first input pipe is open as \( x \) and the aquarium filling time through the second pipe as \( x + 2.5 \). Therefore, 
\[ \frac{V}{x} + \frac{V}{x + 2.5} = 3 \text{ or } 2x^2 - 7x - 15 = 0 \]
This quadratic equation has one positive root equal to 5.

Answer: 5 hours.

8.10. **Solution:** Peter and Ivan spend \( 60/8 = 15/2 \) and \( 20/3 \) minutes, respectively, answering one question. Denote the total number of
questions in the test as $Q$. Then, according to the problem,
\[ \frac{15}{2} \times Q - \frac{20}{3} \times Q = 20. \] The solution of this linear equation is $Q=24$.

Answer: 24.
About the Author:

Victor Rabinovich is a Scientist and an Engineer, with a Ph. D. in Radar Antennas, and is currently tutoring high school and university students. He has more than 40 years of experience in radar, smart antennas and antenna design. Victor published over 40 technical papers in these fields, and holds over 30 Russian and US patents. He also taught course “Mathematical methods in electronics” to university students in Russia, and taught mathematics to advanced high school students.

ACKNOWLEDGMENTS

It’s a pleasure to express my appreciation to the following people who helped me in the completion of this project.

Anna Aleksyuk graduated from Moscow State Pedagogical University. She has more than 40 years of experience as a math teacher in Russian middle and high schools. Her excellent work has been recognized with numerous diplomas of Ministry of Education. She is an Honored Teacher of Russian Federation.

My family:
My family members have provided me with feedback, revisions, and most significantly, support as I worked through getting this workbook published.